Ref. No.: EX/PHY/TE/305/20/2019

M.Sc. (Physics) 2nd Yr, 1st Semester Examination, 2019.

Subject : GENERAL RELATIVITY & ASTROPHYSICS
Paper : PHY/TE/305

Time: Two hours | Full Marks : 40

Answer any four Questions.

1. (a) Let $\mathbf{g} \equiv \det\{g_{\mu\nu}\}$, (where $g_{\alpha\beta}$ denotes the metric tensor). Then discuss the notion of *covariant derivative*, (in terms of the Christoffel connection $\Gamma^{\alpha}_{\beta\sigma}$ etc). Also establish the following results:

$$\mathbf{g}_{,\mu} = (g^{\alpha\beta})\, g\, (g_{\alpha\beta,\mu})$$

(b) In an n dimensional space-time, how many independent components does the metric tensor $g_{\alpha\beta}$ have?

(c) Consider an infinitessimal change (reparametrization) of the coordinates, (generated by a smooth local vector field $\zeta^a(x^\nu)$) of the form:

$$x^a \longrightarrow x'^a = x^a + \zeta^a(x^\mu)$$

Establish that the effect of this coordinate change is to change the metric g_{ab} by :

$$g_{\alpha\beta} \longrightarrow g'_{\alpha\beta} = g_{\alpha\beta} - \zeta_{\alpha,\beta} - \zeta_{\beta,\alpha}$$

Such a change is an example of a *gauge transformation* and is one the basic themes of modern research.

[4 + 2 + 4]

(2). (a) Establish a relation to find out Christoffel connections using metric tensors.

(b) The 2-dimensional *de Sitter* space-time has the line element given by:

$$ds^2 = -du^2 + \cosh^2 u\, d\phi^2, \quad \text{where } -\infty < u < \infty \text{ and } 0 \le \phi < 2\pi.$$

Compute the non-vanishing Christoffel symbols. Can you physically interpret/picturise this Geometry? [4 + (4 + 2)]

(3). (a) Define curvature tensor. Express it in terms of Christoffel symbols and its derivatives. Further from the curvature tensor find out the curvature scalar.

(b). Consider the case of a 3-space (with a positive definite metric). Show that the necessary and sufficient condition for a 3-space to be *flat* is the vanishing of its Ricci tensor. [(1+ 2 + 2) + 5]

(4). Prove the Bianchi identity. Using the Bianchi identity, or otherwise, show that the covariant derivative of the Einstein's tensor vanishes, *i.e.*, $\nabla_\alpha \mathcal{R}_{\alpha\beta} = 0$, where $\mathcal{R}_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$, is the Einstein tensor. [5 + 5]

5. Write down the general form of the Schwarzschild metric tensor for the vacuum solution to Einstein's equations. Starting from the action integral (and using the metric tensor of the space time) find out the Lagrangian and hence obtain the geodesic equations to study the particle dynamics around it. Compare the Schwarzschild's case with the Newtonian/Keplerian scenario. [2 + 6 + 2]