M.Sc. Physics (Day) 2nd Year, 1st Semester Examination, 2019

Dynamical Systems (PHY/TE/202)

Time:Two hours

Full Marks:40

Use separate answer script for each group

Group A

Answer any two questions from Group A

1. (a) Consider a two dimensional linear system described by,

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\,\mathbf{x}\,, \quad \text{where} \quad \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Analyze the equation and classify the fixed points, depending on the determinant $\Delta \doteq \det(\mathbf{A})$ and trace $\tau \doteq \text{trace}(\mathbf{A})$.

(b) Find the fixed points of a nonlinear system described by,

$$\frac{dx}{dt} = -x + x^3\,, \quad \text{and} \quad \frac{dy}{dt} = -2y.$$

Use linearization, and hence study the stability of the fixed points. Show the flow lines in the $x - y$ plane. (6+4)

2. Consider a one dimensional dynamical system given by,

$$\dot{x} = r\,x + x^3 - x^5.$$

(a) Find algebraic expression for fixed point(s) as r varies.

(b) Analyze the stability of fixed points for different r and sketch the vector fields for relevant values of r.

(c) Calculate r_s, the parameter value, at which non-zero fixed points appear in a saddle-node bifurcation.

(d) Find the potential $V(x)$ for the system. Calculate the critical value r_c, such that for $r = r_c$, $V(x)$ has three equally deep minima. Analyze the nature of $V(x)$ when $r \simeq r_c$. (2 +3+1+(1+3))

3. (a) Using a simple generic equation of your choice, explain '*subcritical Hopf bifurcation*'.

(b) In the two dimensional dynamical system given by, $\dot{r} = r(1-r^2)$, $\dot{\theta} = \mu - \sin\theta$, a bifurcation occurs as the value of the parameter μ changes. Find the critical value μ_c and identify the nature of bifurcation.

(c) Obtain the fixed point(s) of the one dimensional map $x_{n+1} = \cos x_n$. By cobweb construction, establish the stability of the fixed point(s).

(3+3+(2+2))

GROUP - B

Answer any TWO questions

1. Find the ratio of the average kinetic energy to the average potential energy of the oscillator under the action of the restoring force given by $F = -kx^3$, where k is a positive constant (the averages should be evaluated over a complete time period).
Marks: 10

2. Consider the dynamical system given by the equations $\dot{x} = y$ and $\dot{y} = -k(x^2-1)y - \omega^2 x$, with $k \gg 1$. Find the approximate time period of the limit cycle.
Marks: 10

3. (a) For the dynamical system given by $\dot{x} = x - y - x(x^2 + 5y^2)$ and $\dot{y} = x + y - y(x^2 + y^2)$, identify a region in phase space that contains a closed orbit and comment on its stability.

(b) For the oscillator $\ddot{x} + A\dot{x} - Bx + x^3 = 0$ with $A > 0$, what kind of bifurcation occurs when the parameter B goes from negative to positive values.

Marks: $5 + 5 = 10$