Ref. No. Ex/PHY/TG/104/20/2019

MASTER OF SCIENCE EXAMINATION, 2019
(1^{st} year, 2^{nd} Semester)
PHYSICS
Statistical Mechanics- I
Paper - PHY/TG/104

Time : Two hours Full Marks: 40

Answer any four questions.

1. (a) Partition function of N independent spins is given by

$$Z = \Pi_{n=1}^{N} \sum_{s_n=\pm 1} e^{-s_n \xi}$$

where $\xi = \mu\beta H$, Find the internal energy and entropy of the system. 2 + 2

(b) Assuming the Dietrici equation of state,

$$P(v-b) = kTe^{-\frac{a}{kTv}}$$

evaluate the critical constants P_C, v_C and T_C of the given system in terms of the parameter a and b. Show that the quantity $\frac{kT_C}{P_C v_C} = \frac{e^2}{2} \simeq 3.695$ 4+2

2. (a) A system consists of two identical, non-interacting, spinless (no spin variables at all) particles. The system has only three single-particle states ψ_1, ψ_2 and ψ_3 with energies $\epsilon_1 = 0 < \epsilon_2 < \epsilon_3$, respectively

(b) List all the two-particle states available to the system, along with their energies, for both Fermions or Bosons. Use the occupation number notation $|(n_1, n_2, n_3) >$ to identify each state. Indicate which state is occupied at T = 0.

(c) Use the Canonical Ensemble to write the partition function for both Fermi and Bose cases.

(d) Using only the leading two terms in the partition function, find the temperature dependence of the internal energy in each case. 3+ 2 + 5

3. (a) In the Ising model, the system considered is an array of N fixed points called lattice size. Associated variable with each lattice site is a spin $\sigma_i (i = 1, 2, ...N)$ which is a number either +1 or -1 , i.e, $\sigma_i = \pm 1$, It is obvious that the energy levels of the system as a whole will be degenerate. Show that the total energy of the system can be expressed in terms of total number of up spin (N_+) and total number of up-up nearest neighbour pair (N_{++}). Hence write down the configurational partition function in this system. 6+3

(b) Besides ferromagnetic system, indicate two other systems which can be modeled using Ising variables. 1

4. (a) Starting from Ising hamiltonian and use Bragg-Williams approxiation, calcuate the partition function in terms of long range order parameter 5

(b) Obtain the expression for ferromagnetic to paramagnetic phase transition temperature in the Bragg-Williams model 5

5. (a) What are the parameters which distinguish one universality class to another ? 2

(b) Give an example of a physical system where one dimentional Ising model can be used to describe its phase transition behaviour. 1

(c) Show that in one dimensional Ising system, there is no phase transition at any finite temperature 7

6. (a) Write down the basic features of Landau's phenomenological theory. Use Landau approach, calculate the critical exponent associated with the order parameter with no external field . . 2 + 4

(b) Starting from equation of state and total number of particles, show that the Fermi system, in general satisfies the relation $P = \frac{2}{3}(\frac{U}{V})$, where U is the internal energy and P, V represent the pressure and volume of the system. 4