Ref. No.: Ex/PHY/TG/103/20/2019

MASTER OF SCIENCE EXAMINATION, 2019

(1st Year, 1st Semester, Day)

PHYSICS

Quantum Mechanics - I

Paper - PHY/TG/103

Time : Two hours **Full Marks : 40**

GROUP - A

Answer ***two*** questions, one from serial no. 1 to 2 and other from serial no. 3 to 4. [2x10=20]

1. a) Find out the general expression of second-order correction to a non-degenerate energy level of a system due to stationary perturbation. b) Calculate the first-order energy correction of nth state of a harmonic oscillator due to perturbation of αx^3.

2. a) Express classically the kinetic energy of a particle moving under a central potential as a sum of radial and rotational kinetic energy. b) Using Hamilton's equation of motion of classical mechanics show that rotational kinetic energy part can be assumed as a potential term when one considers only the radial motion. c) Then find out the radial part of the Schrodinger equation in case of hydrogen atom. Show that the wave function vanishes at $r = 0$. d) Write down the wave function of the $1s$ level of H-atom. e) Find out the expression of first kinetic energy correction term in the Hamiltonian of H-atom if one considers relativistic motion of the electron. Evaluate the size of the ratio of this correction and the non-relativistic kinetic energy.

3. a) Define an observable in Quantum Mechanics. b) Prove that when two observables A and B commute, one can always construct an orthonormal basis of sate space with the eigen vectors common to A and B. c) When a set of observables is called complete set of commuting observables (C.S.C.O.)? d) If the measurement of a physical quantity A on the system in the state $|\psi\rangle$ gives the result a_n, what will be the state of the system immediately after the measurement?

4. a) Show how the matrix elements of an operator transform under change of representation. b) Write down the time evolution equation of the state $|\psi\rangle$ of a system in quantum mechanics. c) Then find the Schrodinger equation in terms of the wave function. d) Show that projection operator is hermitian.

GROUP-B

Answer any Two Questions (2x10=20).

All parts of a question must be written in one place.

(1) (a) Construct the ket $|\vec{S}.\hat{n};+>$ such that $\vec{S}.\hat{n}|\vec{S}.\hat{n};+>=\frac{\hbar}{2}|\vec{S}.\hat{n};+>$ where $\hat{n}$ is characterized by the polar angle β and azimuthal angle α respectively. Treat the problem as a straight forward eigen value problem and express your result as a linear combination of $|+>$ and $|->$ kets. In an alternative method based on rotation operator, verify your result. [5 + 5]

(2) (a) (i)Write down the pauli matrices (σ). (ii) Verify the identity $(\vec{\sigma}.\vec{a})(\vec{\sigma}.\vec{b}) = \vec{a}.\vec{b}+i\vec{\sigma}.(\vec{a}\times\vec{b})$. (iii) Further show that $e^{i\sigma_z\theta} = \cos\theta + i\sigma_z\sin\theta$.

(b) The nth state normalized eigen ket of a harmonic oscillator is given by

$$|n>=\frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0>$$

where all the symbols have their usual meaning. Find the matrix representation of the operator $\hat{a}^\dagger$. [5+5]

(3) Using the angular momentum commutation relations $[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$ and $[J^2, J_k] = 0$, find out the matrix representation of J_i's and $J_\pm$ for $J = 1$. (b) Find the Clebsch Gordan Coefficient for $J_1 = J_2 = 1/2$. [5+5]