Ref. No.: Ex/PHY/TG/101/20/2019

MASTER OF SCIENCE EXAMINATION, 2019

(1st Year, 1st Semester, Day)

PHYSICS

Classical Mechanics - I

Paper - PHY/TG/101

Time : Two hours **Full Marks : 40**

Answer any ***four*** questions.

1. (a) Define allowed displacements and virtual displacements for a constrained system of N particles, obeying ν holonomic, rheonomous constraints given by $f_i(\mathbf{r}_1.\mathbf{r}_2, \ldots \mathbf{r}_N)$; for $i = 1, \ldots \nu$.

   (b) Show by simple diagrams, the allowed and virtual displacements, in the cases of (i) a simple pendulum with support moving vertically downward with velocity $\mathbf{u}$, and (ii) a double pendulum with fixed support.

   (c) Show that in both the above cases the zero virtual work principle holds. ((2+4)+4)

2. Consider a bob of mass m hanging from a fixed support by a *'massless spring'* having spring constant k. The pendulum oscillates in a plane, however its length is not a constant.

   (a) Find the Lagrangian and Lagrange's equations for the system.

   (b) Find the generalised momenta.

   (c) Hence find the Hamiltonian for the system. ((2+3)+2+3)

3. (a) Draw the phase space diagram of a one dimensional harmonic oscillator. Also draw the diagram when a velocity dependent friction is present (you may consider the deceleration to be linear in velocity).

   (b) A particle is executing motion in an attractive central field. Show that its energy and angular momentum are conserved. ((2+2)+(3+3))

4. Two equal masses, m each, are attached to fixed walls and to each other by identical springs, each of spring constant k. The masses execute one dimensional motion on a frictionless horizontal table.

(a) Find the Lagrange's equation for the system.

(b) Find the first and second normal modes of motion.

(c) Give physical interpretation and schematic description of the two modes.

(4+4+2)

5. (a) The Hamiltonian of a one-dimensional simple harmonic oscillator is,

$$\mathcal{H}(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2.$$

Consider a simple transformation $(\mathcal{H}(q,p); q,p) \to \mathcal{H}'(Q,P); Q,P)$,

$$q = \sqrt{\frac{2P}{m\omega}}\sin Q\ , \qquad p = \sqrt{2m\omega P}\cos Q\ , \qquad \mathcal{H}'(Q,P) = \mathcal{H}(q,p).$$

Derive the generating function of the above transformation, and the explicit expression for the transformed Hamiltonian $\mathcal{H}'(Q,P)$.

(b) Define Poisson Bracket. Derive the relation connecting $\{F,G\}_{\{q,p\}}$ and $\{F,G\}_{\{Q,P\}}$, where $\{Q_i, P_i\}$ are the canonically transformed positions and momenta from $\{q_k, p_k\}$. ((2+3)+(1+4))

6. (a) Consider an inertial Lab frame and a rotating frame with common origin. A vector $\vec{r}$ is represented in the two frames as $\vec{r} = \sum_{i=1}^{3} x_i \cdot \widehat{e}_i = \sum_{i=1}^{3} x'_i \cdot \widehat{e}'_i$.

   i. Derive the equations connecting velocities as observed in the Lab and rotating frames.

   ii. Also obtain the equations connecting the corresponding accelerations.

(b) For a rigid body moving with one point fixed,

   i. Write down the rotation matrices $R_x(\alpha)$, $R_y(\beta)$, $R_z(\gamma)$ for rotation by angles α, β and γ about axes x, y and z respectively.

   ii. Define Euler angles. Hence obtain a general rotation matrix R, corresponding to Euler rotations $(\phi,\ \theta,\ \psi)$ about z, x' and z' axes successively.

((2+3)+(1+4))