Ref. No. Ex/M.Sc./M/B1.8/36/2019

MASTER OF SCIENCE EXAMINATION, 2019

(2^{nd} Year, 1^{st} Semester)

MATHEMATICS

Paper – Unit 3.4 (B1.8)

(Computational Fluid Dynamics – I)

Full Marks: 50 Time: Two Hours

The figures in the margin indicate full marks

(Symbols have their usual meaning)

Answer any *Five* questions from the following: 5×10

1. Establish Crank-Nicholson implicit finite difference scheme for the numerical solution of one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \quad \alpha > 0, \quad a < x < b$$

subject to the initial and boundary conditions

$$u(x,0) = f(x), \quad u(a,t) = 0 = u(b,t).$$

Also find the von Neumann stability condition of the scheme.

[5+5]

2. Develop Lax-Wendroff finite difference scheme for solving first order wave equation and hence discuss its Fourier stability analysis. [5+5]

3. Derive alternating direction implicit (ADI) finite difference scheme and discuss its solution procedure of the following partial differential equation

$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ defined over the region $R = \{(x,y) | \, a \le x \le b, c \le y \le d\}, t \ge 0$..

Also give the schematic diagram of ADI method. [8+2]

4. Find the numerical solution of the Dirichlet boundary value problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ defined over a unit square, subject to the following boundary conditions:

$$u(x,0) = u(0,y) = 0; \quad u(x,1) = x, \quad u(1,y) = y$$

by taking $h = k = \frac{1}{3}$ using point successive over relaxation method with $\omega = 1.5$. [10]

5. Explain finite volume method for solving one-dimensional steady state diffusion equation along with its schematic diagram. [10]

6. Consider a problem of source free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100°C and 500°C respectively. Calculate the steady state temperature distribution in the rod with thermal conductivity $k = 1000\ Wm^{-1}K^{-1}$, cross-sectional area $A = 10 \times 10^{-3} m^2$.

[10]

7. Describe SIMPLE algorithm for solving incompressible viscous flow of Newtonian fluid through the discretization of continuity equation and x –momentum equation only.

[10]