(b) Define the "Alexandorff's one point compactification (X^{*}, i_x) of a topological space (X, τ). Prove that the compactification is Hausdorff iff. (X, τ) is locally compact Hausdorff. 5

(c) Among all Haurdorff compactifications of a Tychonoff space, show that Stone-Cech compactification is the largest (or maximal) such compactification. 5

4. (a) If every open cover of a topological space X has a closed locally finite refinement then prove that X is paracompact. 7

(b) Prove that every Hausforff paracompact space is regular. 5

(c) Prove that a complete totally bounded metric space is compact. 4

5. (a) Prove that R^{w} (countable product of R) endowed with the product topology is merizable. 7

(b) State and prove Stone-Weirshass theorem. 1+8

6. (a) Prove that every completely regular space is uniformizable. 8

(b) Show that a uniform space having a countable base is pseudometrizable. 5

(c) Let (X, ν) be a uniform space and let $\hat{\mathfrak{I}}$ be a Cauchy filter in X. If x_0 is a cluster point of $\hat{\mathfrak{I}}$ then prove that $\hat{\mathfrak{I}}$ converges to x_0. 3

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Ex./M.Sc./M/3.1/35/2019

MASTER OF SCIENCE EXAMINATION, 2019

(2nd Year, 1st Semester)

MATHEMATICS

Advanced Topology

Unit - 3.1

Time : Two hours Full Marks : 50

Answer ***q.no. 1*** and any ***three*** from the rest.

1. Define total boundedness and give an example of a metric space which is not totally bounded. 2

2. (a) Prove that x_0 is a cluster point of a net $\{s_n : n \in D\}$ iff. there is a subnet $\{t_\alpha : \alpha \in E\}$ of $\{s_n : n \in D\}$ which converges to x_0. 7

(b) Prove that $f:(X, \tau) \uparrow (y, \tau')$ is continuous at $x_0 \in X$ iff for every net $\{s_n : n \in D\}$ in X converging to x_0, $\{f(s_n) : n \in D\}$ is convergent to $f(x_0)$. 5

(c) For an ultrafilter $\hat{\mathfrak{I}}$, show that if $A \cup B \in \hat{\mathfrak{I}}$, then either $A \in \hat{\mathfrak{I}}$ or $B \in \hat{\mathfrak{I}}$. 4

3. (a) Prove that in a locally compact Hausdorff topological space, every point has a local neighbourhood base consisting of closed compact neighborhoods. 5