Ex./M.Sc./M/B1.13/37/2019

MASTER OF SCIENCE EXAMINATION, 2019

(2nd Year, 1st Semester)

MATHEMATICS

Advanced Differential Geometry and Its Applicaion

Unit - 3.5(B 1.13)

Time : Two hours | Full Marks : 50

Symbols have their usual meanings.
Answer any ***five*** questions.

1. (a) Define differentiable manifold with explanation by a suitable nontrivial example. 2+3

   (b) If x, y, z are differentiable vector fields on a differentiable manifold, then prove that

   $$\left[[x,y],z\right]+\left[[y,z],x\right]+\left[[z,x],y\right]=\theta$$

   what is the name of this identify. 4+1

2. (a) For any r-form ω, show that $d(f^{*}\omega) = f^{*}(d\omega)$ 6

   (b) Define curvature tensor R on a differentiable manifold M and prove that

   $R(fx,y)z = f\ R(x,y)z,\ \forall\, x,y,z\in\chi(M)$ and $\forall\, f \in F(M)$. 4

(Turn over)

3. (a) Prove that in a differentiable manifold which is Hausdorff and second countable has a Riemannian metric. 5

(b) Prove that in a Riemannian manifold

$'R(x, y, z, w) = 'R(z, w, x, y).$ 5

4. (a) Let $\phi : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$\phi(t, (x,y)) = (xe^{2t}, ye^{-3t})$. Check whether ϕ defines a 1-parameter group of transformation on $\mathbb{R}^2$ or not. If so, find its generator. 4

(b) If in a conformally flat Riemannian manifold R(x,y). A = A. R(x,y) holds, where A is a symetric linear transformation, then prove that

$\left(A^2 - \frac{rA}{n-1}\right) x \wedge x = 0$ where r is the scalar curvature of the manifold. 6

5. (a) Define an almost Hermite manifold. Prove that an almost Hermite manifold is Kahler if and only if $\nabla_x \bar{y} = \overline{\nabla_x y}$. 1+3

(b) Show that for a Kahler manifold

(i) $S(\bar{x}, \bar{y}) = S(x, y)$ (ii) $S(\bar{x}, y) + S(x, \bar{y}) = 0$ 4+2

6. (a) Define an almost contact metric manifold an construct an example of it. 4

(b) Prove that in an almost contact metric manifold $g(\phi x, \phi y) = g(x, y) - \eta(x)\,\eta(y)$. 6

7. (a) Prove that in an almost contact Riemannian manifold

$$d\eta(x, y) = \frac{1}{2}\left\{(\nabla_x \eta) y - (\nabla_y \eta) x\right\} \quad 6$$

(b) Prove that $N(x, \bar{y}) = -\overline{N(x, y)}$, where N is Nijenhius tensor in an almost complex manifold. 4

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