

**MASTER OF SCIENCE EXAMINATION, 2019**

( 2nd Year, 2nd Semester )

**MATHEMATICS**

**UNIT – 4.5 ( B - 2.36)**

**EPIDEMIOLOGY AND ECO-EPIDEMIOLOGY**

Time : Two hours

Full Marks : 50

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings)

Answer **questions No 6** and any **Three** from the rest

1. What do you mean by venereal disease? Formulate a mathematical model with necessary assumptions to represent the dynamics of a venereal disease. Find all equilibrium solution of your model system and determine their stability properties. 2+6+8
2. Construct a basic SEIS model with proper assumptions. Draw a schematic diagram of your model. Under what conditions will the epidemic grow? Determine the parametric conditions for which the endemic equilibrium will be locally asymptotically stable. 4+2+2+8
3. What do you mean by travelling wave ? If it is assumed that the disease spreads in one dimensional space with same constant diffusivity for susceptible and infected populations

[ Turn over

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then propose a partial differential equations model for SI type epidemic to obtain travelling wave. Convert your PDE model to equivalent ODE model and determine the critical wave velocity. 2+4+10

4. a) Define eco-epidemiology.  
 b) Consider the following eco-epidemic model :

$$\frac{dP}{dt} = rP \left( 1 - \frac{P}{k} \right) - \alpha P(S + I),$$

$$\frac{dS}{dt} = \alpha \alpha_1 P S - dS - \lambda S I,$$

$$\frac{dI}{dt} = \alpha \alpha_1 P I - (d + \nu) I + \lambda S I.$$

( P = Prey, S = Susceptible predator, I = Infected predator )

- i) Find the basic reproduction number around the disease free equilibrium point of the model. Prescribe the conditions for stability of this equilibrium point. Using Bendixon-Dulac criterion, show that the disease free equilibrium is globally asymptotically stable.
- ii) Show that the existence of positive interior steady state of the model implies its global stability. 3+8+5

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5. a) Formulate an eco-epidemiological model with disease present in prey population with Holling type I functional response with necessary assumptions.  
 b) Determine different steady states of your model.  
 c) Study the local stability of the endemic steady state. 7+4+5
6. Define basic reproduction number with its epidemiological significance. 2