(b) Apply Helmholtz's theorem to decompose the displacement equation, in absence of body forces, of an isotropic elastic body in the form :

$$\nabla^2\phi = \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2}, \nabla^2\vec{\psi} = \frac{1}{c_2^2}\frac{\partial^2\vec{\psi}}{\partial t^2}$$ 7

3. A uniform pressure varying harmonically with time is acting radially on the surface of a cylindrical cavity of radius 'a' in an infinite elastic medium. Determine the displacement at any point of the medium. 15

4. Show that the plane wave solution of the form :

$$\vec{u} = \vec{d}\,F(\vec{r}\,.\vec{N} - ct)$$

to the equation of motion

$$\rho\frac{\partial^2\vec{U}}{\partial t^2} = (\lambda+\mu)\vec{\nabla}\left(\vec{\nabla}\,.\,\vec{U}\right) + \mu\nabla^2\vec{U}$$

is possible if (i) either $\vec{d} = \pm\vec{N}$ or

(ii) $\vec{d}\,.\,\vec{N} = 0$ and $\mu = \rho c^2$ and explain the cases. 15

—— X ——

Ex./M.Sc./M/B1.16/36/2019

MASTER OF SCIENCE EXAMINATION, 2019

(2nd Year, 1st Semester)

MATHEMATICS

Elastodynamics - I

Unit - 3.4(B 1.16)

Time : Two hours | Full Marks : 50

Symbols/Notations have their usual meanings.
All questions carry equal marks.

Answer ***question No. 1*** and any ***two*** from the rest.

1. Show that Rayleigh wave equation have only one real positive root and that for $\lambda = \mu$ the root is $0.9194c_2$. 20

OR

Deduce that for time harmonic plane waves, whether longitudinal or shear total energy density is equally divided between the time averages of kinetic and strain energy densities.

2. (a) Prove that, in an elastic medium, the displacement vector $\vec{F}$ satisfies the vector equation of motion

$$\rho\frac{\partial^2\vec{U}}{\partial t^2} = (\lambda+\mu)\vec{\nabla}\left(\vec{\nabla}\,.\,\vec{U}\right) + \mu\nabla^2\vec{U} + \rho\vec{F}$$

where $\vec{F}$ is the body force. 8

(Turn over)