

5. Using differential geometric structure in phase space , introduce the notion of poisson bracket, show that

$$(i) \vec{X}_H(K) = \{K, H\}$$

$$(ii) [\vec{X}_f, \vec{X}_g] = -\vec{X}_{\{f,g\}}$$

Where K is a function of the phase space. 10

6. What is 'local inertial' co-ordinate system ? If the metric coefficients are independent of any partieuclar co-ordinate, then prove that momentum along that direction is conserved, Show that acceleration vector is Zero along the geodesic.

3+4+3

7. State cosmological principle and weyl postulates. Hence deduce the space time metric for FRW model of the universe.

2+2+6

## MASTER OF SCIENCE EXAMINATION, 2019

( 2nd Year, 2nd Semester )

### MATHEMATICS

#### UNIT – 4.5 (B-2.13)

#### DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS-II

Time : Two hours

Full Marks : 50

The figures in the margin indicate full marks.

Symbols/Notations have their usual meanings

Answer any *five* questions

- Derive the velocity invariant in special theory of relativity. Hence deduce the velocity composition law. Show that  $v \oplus \lambda = \lambda$ ,  $\lambda \oplus \lambda = \lambda$  4+3+3
- Write down Einstein-Hilbert action. Derive Einstein equations from this action. 2+8
- What do you mean by killing vector fields ? Give an example of a killing vector field. When is a space said to be homogenous ? When is a space said to be maximally symmetric ? How many distinct killing vector fields are there for a maximally symmetric spacece ? 3+2+2+2+1
- Define Einstein tensor. Prove that the divergence of the Einstein tensor vanishes. What is the physical interpretation of this relation ? How many independent field equations are there in four dimensional space time ? What are the constrain equations ? 1+3+2+2+2

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