M. Sc. Mathematics Examination, 2019

(1st Year, 2nd Semester)

Mathematics

Ordinary Differential Equations and Special Functions

Unit - 2.5

Time : Two hours Full Marks : 50

The figure in the margin indicate full marks

Symbols have their usual meanings

PART-I (25 marks)

Answer *Q.No.1* and *any two* from the rest :

1. a) State the existence and uniqueness theorem of the initial value problem of the ordinary differential equation.

   b) If a= a(x), b-b(x) and c=c(x) are continuously differentiable functions of x with $a \neq 0$ and $L = a\frac{d^2}{dx^2} + b\frac{d}{dx} + c$ is not self-adjoint operator, find $\mu = \mu(x)$ such that μL is self-adjoint.

   2+3

2. a) If the homogeneous part of the equation

$$x^3\frac{d^2y}{dx^2} - x^2(2x+1)\frac{dy}{dx} + x^2(x+1)y = 2e^x$$

has a solution of exponential form then find the general solution of the given equatuion.

b) Prove that the differential operator corresponding to the Lagendra equation of order n (real) is self-adjoint. 7+3

3. a) (i) If $\upsilon_1 \upsilon_2$ are the solutions of the differential equation :

$p\frac{d^2u}{dx^2}+q\frac{dy}{dx}+ru=0,$ write the Green's finction for the following boundary value problem : (x, ε)

$$Lu = p\frac{d^2\upsilon}{dx^2}+q\frac{dy}{dx}+ru=f(x), 0<x<1$$

Subject to the boundary conditions : $h_1u(0)+h_2u'(0)=0$

and $H_1u(l)+H_2u'(l)=0$, where

$h_1^2+h_2^2 \neq 0, H_1^2+H_2^2 \neq 0$ and f is continuous in [0,1] along with the appropriate conditions on p=p(x), q=q(x) and r=r(x).

(ii) Use the difinition of G to prove that $G(x, \varepsilon)$ to prove that $G(\varepsilon+0, \varepsilon)=G(\varepsilon-0, \varepsilon)$.

b) Find the green's function of the following boundary value porblem

$$\frac{d^2y}{dx^2}-\alpha^2u=f(x), 0<x<1, \alpha>0$$

subject to the boundary conditions : $u'(0)=0$ and $u'(l)=0$, where f is continuous in [0, 1]. (2+1)+7

4. a) Let f and g be two solutiuons of

$$\frac{d}{dt}\left[p(t)\frac{dx}{DT}\right]+\phi(T)X=0$$

Where f and g have a commion zero on the interval $a \le t \le b$, p is differentiable in (a, b) and is continuous in [a, b]. Prove that *f* and *g* are linearly dependent on $a \le t \le b$.

b) If, however, f and g be two real linearly independent solutions of the above equation then show that there is precisely one zero of g between any two consecutive zeros of f. 4+6