M. Sc. Mathematics Examination, 2019

(1st Year, 2nd Semester)

Mathematics

Mechanics of Continua

Unit - 2.4

Time : Two hours Full Marks : 50

The figure in the margin indicate full marks

Symbols have their usual meanings

Answer *any five* questions :

1. a) Deduce Cauchy's formula involving the stren vector components and stren tensor components.

   b) If the state of stress at any point of a body be given by

   $T_{xx}= y^2+() (x^2–y^2)$,

   $T_{yy}=x^2+ () (y^2–x^2)$

   $T_{zz}=x^2+y^2$,

   $Tyz=T_{zx}=0$ and $Txy=f(x, y)$

   Determine the expresion for T_{xy} in order that the stren distribution is in equilibrium in the absence of body forces. 5+5

2. Define principal stress abd principal strain directions. Show that all the three principal stren value are real and, if they

are distinet then show that the corresponding principal stress direction are mutually perpendicular. 10

3. a) Show that the fundamental meadure of deformation of a body at a point (x, y, z) is given by

$$2\left(E_{xx}dx^2 + E_{yy}dy^2 + E_{zz}dz^2 + 2E_{zy}dxdy + 2E_{yz}dydz + \right.$$

$$\left. +2E_{xx}\,dzdx\right)$$, Where Eij (i, j=x, y, z) are Lagrangian Finite strain tensor at (x, y, z).

Whay do you mean by infinitesimal strain ? Deduce the exprenians for eij (i.j=x,y, z) for infinite simal strain components.

b) The diaplacement in an elastic solid in given by

$$x_x = a\left(x + 2y + 3z\right), x_y = a\left(-2x + y\right)$$

$$u_z = a\left(x + 4y + 2z\right)$$, Where a is a small quantity

Find the infinitesimal strain tensor and the cubical dilatation. 6+4

4. a) Give the geometrical interpretation of longitudinal strains.

b) What do you mean by volumatric strain ? Show that the Volumetric strain is euqal to the sum of the three longitudinal strains. 5+5

5. Derive the stress equation of motion of a contineum, appling

the principle of linear momentum, stence deduce the Euler's equation of motion for a perfect fluid. 10

6. Deduce the Navier-stokes equation of motion of a () compressible fluid in the following form :

$$\frac{\partial\vec{q}}{\partial t} - \left[\vec{q} \times \text{curl}\vec{q}\right] = \vec{F} - \text{gradQ} + \frac{4}{3}\delta \text{ grad div } \vec{q} - \delta \text{ ceerl ceerl } \vec{q}$$

10

7. By applying the principle of angular momentum, show that the stress tensor is symmetric. 10