(b) In a dynamical system with two degrees of freedom, the K. E. and P. E. are given by where

$$T = \frac{\dot{q}_1^2}{2\left(a + \dot{b}q_2\right)} + \frac{1}{2} q_2^2\, \dot{q}_2^2, V = c + dq_2$$

where a, b, c and d are constants. Show that the value of q_2 in terms of time is given by the equation of the form

$$(q_2 - k)\,(q_2 + 2k)^2 = h(t - t_0)$$

with h, k and t_0 as constants. 5+5

7. (a) Establish Hamilton's principle from D'Alembent's principle.

(b) Two heavy uniform rods AB and AC each of mass M and length 2a are hinged at A and placed symmetrically over a smooth cylinder of radius 'c' whose axis is horizontal. If they are slightly and symmetrically displaced from the position of equilibrium, show that the time of small oscillation is

$$2\pi \sqrt{\frac{a \sin\alpha}{3g}\left(\frac{1+3\sin^2\alpha}{1+2\sin^2\alpha}\right)}$$

where $a \cos^3\alpha = c \sin \alpha$. 5+5

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Ex./M.Sc./M/1.4/32/2019

MASTER OF SCIENCE EXAMINATION, 2019

(1st Year, 1st Semester)

MATHEMATICS

General Mechanics

Unit - 1.4

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks
(Symbols have their usual meaning)

Answer any ***five*** questions.

1. (a) State the Lioville's form for a dynamical system.

(b) The K. E. and P. E. of a particle moving in a plane are given by

$$2T = \dot{x}^2 + \dot{y}^2, V(r) = -\frac{\mu}{r} - \frac{\mu^1}{r^1}$$

where r, r^1 are the distances of the particle from the points (c,0) and (–c,0) respectively. Show that the problem can be formulated in Lioville's form. 3+7

(Turn over)

2. Show that the K. E. of a moving system can be expressed as

$$T = \frac{1}{2}\sum_{i,j} a_{ij}\ \dot{q}i\,\dot{q}j + \sum_{i} b_i\ \dot{q}i + C$$

If for a scleronomic system $T^{/}(q,p)$ is what T becomes when expressed in terms of variables q and p, then prove that

(i) $\dot{q}i = \frac{\partial T^{/}}{\partial pi}$ (ii) $\frac{\partial T}{\partial qi} + \frac{\partial T^{/}}{\partial qi} = 0$

(iii) $T^{/}$ is a homogeneous quadratic in p's.

(iv) $T + T^{/} = \sum p_i \dot{q}_i$.

(v) $2T^{/} + \frac{D}{\nabla} = 0$ where $\Delta = \det(a_{ij})$ and

$$D = \begin{vmatrix} a_{11} \ldots\ldots a_{1n}\ p_1 \\ a_{21} \ldots\ldots a_{2n}\ p_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1} \ldots\ldots a_{nn}\ p_n \\ p_1 \ldots\ldots\ldots p_n\ \ 0 \end{vmatrix}$$

3+1+1+1+1+3

3. (a) Define action of a mechanical system. State and prove principle of least action.

(b) What do you mean by Legendre's dual transformation ?

(c) Derive Hamilton–Jacobi partial differential equation. (1+3)+2+4

4. (a) Define canonical transformation. Show that Poisson bracket remains invariant under canonical transformation.

(b) Show that the transformation

$$Q = \sqrt{q}\ \cos p, P = \sqrt{q}\ \sin p$$

represents a canonical transformation Hence evaluate the new Hamiltonian of the system for which

$$T = \frac{1}{2} m\dot{q}^2\ , V = \frac{1}{2} kq^2$$ (1+4)+(3+2)

5. (a) Show that $J_1 = \int_{S_2} \sum d\, q_i\, dp_i$, is invariant under canonical transformation where S_2 is a 2D surface in phase space.

(b) Establish Euler's dynamical equations from the principle of angular momentrum. 5+5

6. (a) Establish Earth's rotation using Foucault's pendulum.