M. Sc. Mathematics Examination, 2019

(1st Year, 2nd Semester)

Mathematics

Functional Analysis

Unit - 2.3

Time : Two hours Full Marks : 50

(25 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer Q. No. ***1*** and ***any four*** from rest.

(Unexplained symbols and Notations have their usual meaning)

1. Justify any five of the following : 1×5=5

   a) Every reflexive space is a Banach space.

   b) $C_O^{**} = l_\infty$ implies that C_O is not reflexive.

   c) The identity operator on an infinite dimensional normed linear space is not compact.

   d) Let M be a normed linear space and N a closed subspace of M. Then the canonical linear transformation.

   $T : M \to M/N,\ V \to v + n,$ is bounded with $\| T \| \le 1.$

e) The geometric dual and the algebraic dual of a finite dimensional normed linear space are equal.

f) There exist linear operator on a normed linear space which has spectral value but does not have any eigenvalue.

2. State and prove Riesz Lemma. 1+4

3. a) Prove that if the closed unit ball in a normed linear space N is compact then N is finite dimensional.

   b) State Open Mapping theorem. 4+1

4. a) Using open mapping theorem prove that if N_1 and N_2 are banach spaces then the invrse of a bijective continuous linear transformation $T : N_1 \to N_2$ is continuous.

   b) State Hahn Banach Theorem for normed linear spaces.

   Prove that if N is a non zero normed linear space and $v(\neq 0) \in N$ then these exists a bounded linear functional $f \in N^*$ such that $f(\upsilon) = \|\upsilon\|$ and $\|f\| = 1$. 2+3

5. Let N be a real or complex vector space. Then prove that for two norms. $\|\cdot\|_1$ and $\|\cdot\|_2$ on N the following are equivalent.

   i) $\|\cdot\|_1$ and $\|\cdot\|_2$ generate the same topology.

   ii) these exist $k_1 > 0$, $k_2 > 0$ such that $k_1 \| v \|_1 \leq \| v \|_2 \leq k_2 \| v_1 \|$ for all $v \in N$. 5

6. State Uniform Boundedness principle. Prove that a non empty subset X of a normed linear space N is bounded if and only if f(X) is a bounded set of scalars (real or complex numbers) for all $f \in N^*$. 1+4

7. Let N be a normed linear space and $T : N \to N$ be a linear transformation. Define the transpose T' of T. Prove that $T'(N^*) \subseteq N^*$ if and only if T is continuous. In that case if the restriction of T' to N^* is denoted by T^* then prove that $\|T\| = \|T^*\|$. 3+2