Ex./M.Sc./M/1.5/32/2019

MASTER OF SCIENCE EXAMINATION, 2019

(1st Year, 1st Semester)

MATHEMATICS

Differential Geometry

Unit - 1.5

Time : Two hours Full Marks : 50

Symbols/Notations have their usual meanings.
All questions carry equal marks.

Answer any ***five*** questions.

1. (a) Find the cylindrical coordinates and spherical coordinates of the point whose rectangular Cartesian coordinate is $(1, \sqrt{3}, 1)$. 4

(b) Let $A = (2,1,1)$ be a contravariant vector in a coordinate system (x^1, x^2, x^3) of $\mathbb{R}^3$. Find its components in the coordinate system $\left(\bar{x}^1, \bar{x}^2, \bar{x}^3\right)$. 4

(c) Where $\bar{x}^1 = 3x^1 - 3x^2 + 2x^3$, $\bar{x}^2 = 2x^2 + 3x^3$, $\bar{x}^3 = x^1 + x^2 + 2x^3$, Prove that $\begin{Bmatrix} h \\ i\ j \end{Bmatrix} = \begin{Bmatrix} h \\ j\ i \end{Bmatrix}$. 2

(Turn over)

2. (a) Assume that $X(i,j)B^j = C_i$ holds, where B^i is an arbitrary contravariant vector and C_i is a covariant vector. Show that X(i,j) is a tensor. What is its type? 3+1

(b) In V^2, find the quantities g^{ij}, if $g_{ij} = i+j$. 2

(c) When is a vector in a Riemannian space said to be a unit vector? Check whether the contravariant vector with components $\left(1,0,0,\sqrt{2}/c\right)$ is a unit vector or not in a space with line element $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$, c being a constant. 1+3

3. (a) Show that $A^i{}_{,i} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}\left(A^i\sqrt{g}\right)$, $g = |g_{ij}| \neq 0$.

What is the geometrical interpretation of $A^i{}_{,i}$. 6

(b) Define curvature tensor on a Riemannian manifold. State and prove first Bianchi's identity regarding the curvature tensor. 4

4. (a) Find the curvature at any point of the curve $C : x^1 = a, x^2 = t, x^3 = ct$, where $ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (dx^3)^2$, a, c are nonzero constants. 5

(b) Define Bertrand mates in a space. Prove that for Bertrand mates the curvature k and torsion τ are related by $ak + b\tau = 1$, whre a, b are nonzero constants. 5

5. (a) Prove that the necessary and sufficient condition for a given curve to be a helix is that the ratio of the curvature to the torsion is constant. 5

(b) Find the differential equation of the geodesic on a surface with metric

$$ds^2 = (du)^2 + (v^2 - u^2)(dv)^2$$ 5

6. (a) Prove that at each non-umbilical point of a surface there exist two mutually orthogonal directions for which the normal curvature attains its extreme values. 6+4

(b) Find the equation for the principal curvatures of the surface for the right helicoid.

7. (a) Prove that $a^{\alpha\beta} c_{\alpha\beta} = 4H^2 - 2K$. 3

(b) Find the mean curvature of the right helicoid and explain the result by geometrical interpretation. 6

(c) Write the relation between space unit tangent vecor and surface unit tangent vector. 1

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