M. Sc. Mathematics Examination, 2019

(1st Year, 2nd Semester)

Mathematics

Topology

Unit - 2.2

Time : Two hours Full Marks : 50

Answer Question No. ***1*** and ***any three*** from rest

1. On an infinite set X, define a topology z so that X has no limit point and also define a topology τ_1 so that every point of X is a limit point of X. 2

2. a) Define a neighborhood operator ν on X. Show that given a neighborhood operator v on X, ther exists a topology z on X such that $\nu(x)$ coincides with N_x = the system of all neighborhoods of x. 8

   b) Prove that $f:(X,\tau)\to(Y,\tau')$, where (X, z) and (Y,τ') are topological spaces, is continuous iff for any $B\subset Y$, $f^{-1}(\overline{B})\supset\overline{f^{-1}(B)}$. 3

   c) Let (x,τ) be a topological space and $Y\subset X$. Define the subspace topology τ_y. Prove that for any set $S\subset Y$, $\overline{S}\,\tau_y=\overline{S}_\tau\cap Y$. 5

3. a) Prove that an infinite Havsdorff topological spae contains an infinite sequence of pairwise disjoint open sets. 6

b) Prove that a topological space is completely normal iff every subspace of it, is normal. 6

c) Prove that every second countable topological space is Lindeloff. 4

4. a) Prove that the union of a family of connected subsets in a topological space, none of which are separated is also connected. 5

b) Prove that every continuous function $f : [0, 1] \rightarrow [0, 1]$ must have a fixed point. 5

c) Give an example of a topological space which is connected but not path-connected. 3

d) Prove that the components in a topological space are closed and pairwise disjoint. 3

5. a) State and prove Alexander Subbase Theorem. 3

b) Use it to prove that arbitary product of compact spaces endowed with product topology is also compact. 7

c) Prove that a topological space (x, τ) is countably compact iff every infinite sequence in x has a cluster point. 6

6. a) For any transfinite cardinal number α, prove that

i) $\alpha + n = \alpha$

ii) $\alpha + \alpha = \alpha$ 3+6

b) Define an ordinal number. prove that for any two ordinal numbers and either $\beta < \nu$ or $\beta = \nu$ or $\beta > \nu$. 7