MASTER OF SCIENCE EXAMINATION, 2019

(1st Year, 1st Semester)

MATHEMATICS

Real Analysis

Paper : 1.2

Time : Two hours Full Marks : 50

Answer ***q.no. 1*** and any ***three*** from the rest.

1. Give an example of a continuous function which is not a function of bounded variation. 2

2. (a) Prove that the Lebesgue outer measure of an interval is equal to its length. 7

   (b) Define a Lebesgue measurable set. If E and F are Lebesgue measurable show that E-F and E∪F are also so. 1+4

   (c) If $E \subset [0,1\rangle$ and $x \in [0,1\rangle$ then prove that $E \dotplus x$ is measurable if E is so with

$$\mu\left(E \dotplus x\right) = \mu(E\rangle$$ 4

3. (a) Define a ring R. For a class of sets E if R(E) is the ring generated by E then prove that R(E) is countable if E is countable. 7

(Turn over)

(b) Is the above result true for the σ-ring generated by E ? Justify your answer. 2

(c) If f is measurable and f = g a.e. then prove that g is also mesurable. 3

(d) If $f_n \to f$ in ⓜ and $f_n \to g$ in ⓜ then show that f = g a.e. 4

4. (a) Give two examples to show that in general convergence in measure does not imply pointwise convergence the converse is also not ture. 6

(b) State and prove Egoroff's Theorem. 10

5. (a) Prove that a bounded function f defined on a measurable set E of finite measure is Lebesgue integrable on E iff f is measurable. 7

(b) For a sequence of non-negative measurable functions $\{f_n\}_n$ defined on a measurable set E show that $\int_E \sum_n f_n \, d\mu = \sum_n \int_E f_n \, d\mu$. 4

(c) For a function f defined on [a,b] and a < c < b, prove that f is a function of bounded variation on [a,b] iff f is a function of bounded variation on [a,c] and [c,b] and $\bigvee_a^b f = \bigvee_a^c f + \bigvee_c^b f$. 5

6. (a) State and prove Dominated Convergence Theorem. 6

(b) Let f be a non-negative function which is Lebesgue integrable on a measurable set E. Then prove that for $\in > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $\mu(A) < \delta$, we have $\int_A f d\mu < \in$. 5

(c) Let f be Lebesgue integrable on [a,b]. Then prove that $\int_A^x f(t)dt = 0$ for all x $\in$ [a,b] iff f = 0 a.e. on [a,b]. 5

—— X ——