13. Two groups A and B, consist of 100 people each who have a disease. A serum is given to group A but not to group B (which is called control group) ; otherwise, the two groups are treated identically. It is found that in groups A and B, 75 and 65 people, respectively recover from the disease. Test the hypothesis that the serum helps to cure the disease using a level of significance of (a) 0.01 and (b) 0.05 using the chi-square test.

a) Construct the contingency table

b) Complete the four steps of the hypothesis test

c) Decide whether H0 would have been rejected or accepted with the following significance level

(i) $\alpha = 0 \cdot 01$, (ii) $\alpha = 0 \cdot 05$.

(Given $\chi^2_{1,\,0\cdot05} = 3 \cdot 84$, and $\chi^2_{1,\,0\cdot01} = 6 \cdot 63$) 2+8+6

Cleanliness - 2

M. Sc. Bio-Technology Part I Examination, 2019

Mathematics & Biostatistics

Paper - MSBT 1/6

Time : Four hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions. 10×5=50

1. a) Prove that (without expanding) 3

$$\det \begin{matrix} a & b & c \\ x & y & z \\ p & q & r \end{matrix} = \det \begin{matrix} y & b & q \\ x & a & p \\ z & c & r \end{matrix}$$

b) Calculate the determinant (without expanding) 3

$$\begin{matrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{matrix}$$

c) Prove without expansion that 4

$$\det \begin{matrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{matrix} = 0$$

2. a) If A^{-1} and B^{-1} exist, where A and B are both square matrices of order n×n then show that, $(AB)^{-1} = B^{-1}A^{-1}$ 3

b) What do you mean by skew – symmetric matrix? Give one example. 2

c) If the Matrix $A = \begin{matrix} 3 & 1 \\ -2 & 2 \end{matrix}$, show that A satisfils

$A^2 - 5A + 71 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence deduce the inverse of A (1 = identity matrix) 3+2

3. a) Verify Cayley–Hamilton theorem for the matrix

$$A = \begin{matrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{matrix}$$ 6

b) Show that, the matrix $S = \begin{matrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{matrix}$ is an orthogonal matrix. If $P = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$,

Show that SPS^T is a diagonal matrix 4

4. a) Solve the equations by matrix method

$x + 2y + 3z = 14$

$2x - y + 5z = 15$

$2y + 4z - 3x = 13$ 5

c) Decide whether H_o would have been rejected or accepted with the following significance level ;

(i) $\alpha = 0 \cdot 01$, (ii) $\alpha = 0 \cdot 05$.

d) Considering the result of the hypothesis test, decide which of the Type I or Type II errors is prossible and explain.

2+8+4+2

12. In Mendel's experiments with peas he observed 315 round and yellow, 108 round and green, 101 wrinkled and yellow, and 32 wrinkled and green. According to his theory of heredity the numbers should be in the proportion 9:3:3:1. Is there any evidence to oubt his theory at the (a) 0.05, (b) 0.01 level of significance ?

(Given $\chi^2_{3,\,0\cdot01} = 11 \cdot 3$ and $\chi^2_{3,\,0\cdot05} = 7 \cdot 81$)

a) Explain how the data for this hypothesis test is appropriate for a χ^2 test.

b) Complete the four steps of the hypothesis test.

c) Decide whether H_O would have been rejected or accepted with the following significance level :

(i) $\alpha = 0 \cdot 01$, (ii) $\alpha = 0 \cdot 05$. 2+8+6

8. a) Evaluate $\int_{10^o}^{22^o} \log(\cos ax + \sin bx)dx$ where $a = 1, b = 6$ by Trapezoidal Rule for 12 intervals. 5

   b) Prove that $F_s[xf(x)] = -\frac{d}{dw}F_c(w)$ 2

   c) What do you mean by integral transform of a function ? Write down the 'Kernel' of a Laplace Transform of a function ? 2+1

9. a) Find the Fourier Transform of the function

$$f(x) = \begin{cases} 1-x^2 & \text{for } \text{mod.}(x) \le 1 \\ 0 & \text{for } \text{mod.}(x) > 1 \end{cases}$$ 5

   b) Evaluate $\int_1^3 \sqrt{(5x^2 + 4x + 11)}\, dx$ by Simpson's one third rule (N = 12) 5

PART - II

Answer ***any three*** questions. 16×3

10. Score in a particular class are surveyed. Each respondent is assigned an identification number, and information about each of the following is recorded : sex of the students and Score

Sex	Score
M	35
F	20
F	35
M	41
M	39
M	59
F	20
M	52
F	44
M	46
F	40
F	34
F	24
M	62
M	44

a) Complete the construction of figure to obtain a stem-and-leaf display for males and a stem-and-leaf display for females.

b) Obtain an ordered array for males and an ordered array for females.

c) Find the five-number summary for males and the five-number summary for females.

d) Construct a box plot for males and a box plot for females.

e) Does the distribution of "Age" appear to be centred at a different value for males and females ? If yes, for which sex does the centred of the distribution appear to be greater ? 4+2+4+4+2

11. A principal of a private school claims that the students in his class are above average intelligence. A random sample of nine students IQ scores are : 89, 99, 105, 116, 116,118, 119, 125 and 128. Is there sufficient evidence to support the principal's claim ? The average IQ of all the students of this school is 100 with a standard deviation of 15.

(Given $t_{8\,;\,0.05} = 1.860$ and $t_{8\,;\,0.01} = 2.896$)

a) Explain how the data for this hypothesis test is appropriate for a Student's t test.

b) Complete the four steps of the hypothesis test

b) Solve the following system of equations by Cramer's rule $2x - y = 3\,;\ 3y - 2z = 5\,;\ 2z - x = -4$ 5

5. a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{matrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{matrix}$$ 3+4

b) State Gauss' divergence theorem and express it in a mathematical form. 2+1

6. Solve the following differential equation ***(any two)*** 2×5

i) $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

ii) $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$

iii) $y \sin 2x dx - (1 + y^2 + \cos^2 x) dy = 0$

7. Solve the following differential equations ***(any two)*** 2×5

a) $D^2y - 3Dy + 2y = x^3$

b) $D^2y - 9y = e^{3x} \cos x$

c) $(D^2 - D - 2)y = \sin 2x \quad \leftrightarrow \quad \left(D = \frac{d}{dx}\right)$