Ref.Ex: FP/H 09/GR-A/36/2019

BACHELOR OF SCIENCE FINAL EXAMINATION, 2019

(3rd Year, 1st Semester)

PHYSICS (Honours)

Time : 2 hrs. Paper- HO-9 Full Marks : 50

(25 marks for each group)

GROUP -A

Use separate Answer Scripts for each group

Answer question 4 and any two questions from the rest.

1. (a) The equation of motion of a particle under the impulsive force in a viscous medium is given by
$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = P\delta(t)$. Solve the equation with the help of Laplace transformation with initial condition $x'(0) = x(0) = 0$.
(b) Show that $\nabla^2\frac{1}{|\vec{x}-\vec{x_0}|} = -4\pi\delta^3(\vec{x}-\vec{x_0})$ where $\vec{x}$ is a position vector.
(c)State and prove the Parseval's theorem in connection to the Fourier Transformation. [4+3+3]

2.(a)Let $f(x,t)$ be a solution of the heat equation $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at $t = 0$ is $f(x,0) = e^{-x^2}$ for $-\infty < x < \infty$. Then for all $t > 0$, find $f(x,t)$ using Fourier Transformation technique.
(b)Prove the following
$\cos x = J_0(x)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}(x)$, and $\sin x = 2\sum_{k=1}^{\infty}(-1)^{k-1}J_{2k-1}(x)$. [5+5]

3. (a) Let $f(z)$ be analytic in a region R bounded by two concentric circles C_1 and C_2 and on the boundary. If z_0 be any point in R then find $f(z_0)$.
(b) Prove that $\int_0^{2\pi}\cos^{2n}\theta d\theta = 2\pi\frac{1}{2^{2n}}\frac{2n!}{n!n!}$.
(c) If $f(z) = (z^2-1)^{1/2}$, find the branch points and branch lines. Show that along the branch lines phases are not same and differ by π.
(d) With $z = x + iy$, which of the following is an analytic function of the complex variable z in the domain $|z| < 2$?
(i) $(3+x-iy)^7$, (ii) $(1+x+iy)^4(7-x-iy)^3$
(iii) $(1-2x-iy)^4(3-x-iy)^3$ $(iv)(x+iy-1)^{1/2}$. [2+3+3+2]

4. Evaluate the following integral using complex analysis:
In quantum mechanical analysis of scattering leads to the function

$$I(\sigma) = \int_{-\infty}^{\infty}\frac{x\sin x}{x^2-\sigma^2}dx$$

where σ is real and positive. This integral is divergent and therefore ambiguous. From the physical conditions of the problem there is a further requirement $I(\sigma)$ is to have the form $e^{i\sigma}$ so that it will represent an outgoing scattered wave. [5]

OR

$I = \int_0^{\infty}\frac{x^{p-1}}{1+x}dx$, for $0 < p < 1$. [5]

Group B (25 Marks)

Statistical Mechanics - I

Answer **any five** questions (all carry equal marks) :

1. (a) Define the probability density function (p.d.f.) $f_X(x)$ of a continuous variable.

   (b) Does the p.d.f. have any unit or dimension ? Justify.

   (c) Let $f_X(x)$ be the p.d.f. of a random variable X. If we transform the random variable as, $Y = F(X)$, where F is a continuous function defined everywhere, not necessarily invertible, what is the p.d.f. $f_Y(y)$ of Y ?

   (d) **Show how the Fourier transform of the p.d.f., i.e. $\tilde{f}(k)$ can generate the** moments of the p.d.f. via an appropriate expansion. (1+1+2+1)

2. (a) The phase space of a one dimensional single particle is described by its coordinate q and momentum p. Consider a rectangular phase space volume between $q = q_1$, $q = q_2$, $p = p_1$ and $p = p_2$. Show that for a free, non-interacting particle, the phase space volume remains invariant in time.

   (b) State Liouville's Theorem. Argue its connection to the above problem. (3+2)

3. Consider a system of N one dimensional distinguishable classical harmonic oscillators having equal angular frequency 'ω'. In the micro-canonical ensemble, derive an expression for the number of micro-states of the system confined within an energy hyper-surface of radius E. Clearly mention approximations and their justifications, if any. (5)

4. A classical ideal gas with N identical particles is described by the Hamiltonian

$$H = \sum_{k=1}^{N} \frac{|\vec{p}_k|^2}{2m}.$$

(a) Find the canonical partition function and the entropy.

(b) Discuss briefly Gibbs' paradox in relation to this problem. Why is this called a "*paradox*" ? (3+2)

5. Consider an ultra-relativistic ideal gas in 3 spatial dimensions described by the Hamiltonian,

$$H = \sum_{k=1}^{N} |\vec{p}_k| c.$$

Find the canonical partition function and the entropy. (3+2)

6. Let us consider a linear chain of N 'spins' each of which can take integer values, $S_k = \pm 1$. Where $(+1)$ represents an up-spin and (-1) represents a down-spin, and suffix 'k' indicates location on the linear chain. The energy of the system in absence of external magnetic field is given by,

$$E_{\text{Ising}}\{S_j\} = -J \sum_{<j,k>} S_j\, S_k\,, \qquad J \geq 0.$$

Where summation runs over nearest neighbours.

(a) For the one dimensional chain having the topology of a circle (system with periodic boundary condition), find transfer matrix and partition function. Mention clearly any assumption(s) made to arrive at the result.

(b) Derive by exact calculation, an expression for magnetisation per spin. (2+3)