(a) Show that for very small p(p << 1) and hence n << N, ω(n) transforms into a Poisson distribution $\omega(n)=\frac{\lambda^n}{n!}e^{-\lambda}$, where λ = Np is the mean number of events. 3

(b) Show that for large N, the above expression for ω(n) leads the to Gaussian probability distributions. 7

11. (a) Explain the concept of phase space in classical statistics ? 2

(b) Calculate the number of accessible microstates for a classical simple harmonic oscillator in the energy range E and E + δE. 3

(c) Draw the phase space diagram of the simple harmonic oscillator whose energy lies between E and E + δE. 1

(d) Consider a quasi-static process in which the system A, by virtue of its interaction with system A$^{/}$, is brought from an equilibrium state, described by energy E and an external parameter x, to an infinitesimally different equilibrium state described by E + dE and x + dx. What is the resultant change in the number of states Ω accessible to A ? Hence show that using first law of thermodynamics, $ds=\frac{dQ}{T}$, where symbols have their usual meaning. 4

Ex/UG/SC/CORE/PHY/TH/09/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(2nd Year, 2nd Semester)

PHYSICS

Mathematical Methods - III and Statistical Mechanics - I

Paper : CORE - 9

Time : Two hours Full Marks : 50

Use separate answer scripts for each group.

GROUP - A (25 marks)

(Mathematical Methods - III)

Answer any ***five*** questions.

1. Starting from the fact that the volume of a D-dimensional hypershere is proportional to the D-th power of its radius, find the exact expression for the volume. 5

2. Show that

$$\int_0^{\pi/2}\cos^{2m-1}\theta\sin^{2n-1}\theta\,d\theta=\frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$$

3. Find the expression for the divergence $(\vec{\nabla}.)$ operator in terms of the generalized curvilinear coordinates q_1 in three dimensions. 5

4. Show that the norm of a well-behaved function is the same as the norm of its Fourier transform. 5

(Turn Over)

5. Find the Fourier series expansion of the function f(x) given as f(x) = 0, for $-\pi \le x \le 0$ and f(x) = x for $0 \le x \le \pi$, and hence show that $\frac{\pi^2}{8} = 1 + 1/9 + 1/25 + \dots$. 5

6. Show that the differential equation $6x^2y'' + x(1+6x)y' + y = 0$, (where primes denote derivative with respect to x) has two independent series solutions [you need not find the full series solutions]. 5

GROUP - B

(Statistical Mechanics - I)

Answer ***q.no.*** *7* and any ***two*** from the rest.

7. Consider a paramagnetic material consisting of a volume V of N non-interacting spin-1 particles with magnetic dipole moment $\vec{\mu} = \frac{\mu}{\hbar}\vec{S}, S_z = m\hbar, m = 0, \pm 1$ that are located in a magnetic field $\vec{B} = B_0\vec{z}$ at temperature T.
   (a) Write the partition function in terms of $c = \mu H_0$
   (b) Compute the average spin $< S_z >$ 3+2

8. Consider system A and A' are initially sperately in equilibrium and isolated from each other. Now systems are brought into contact with each other and allow only thermal interaction between them. Use microcanonical ensemble to

   (a) establish the statistical defination of temperature and entropy. 4
   (b) show that heat flows from a system at higher temperature to a system at lower temperature. 3
   (c) show that total entropy of the composite system always increases. 3

9. (a) Starting from the general expression for entropy $(S = -k < \ln P_r >)$, derive the expression for entropy of a canonical ensemble. 3
   (b) A gas of N indistinguishable classical non-interacting atoms is held in a neutral atom trap by a potential of the form V(r) = ar where $r = \sqrt{x^2 + y^2 + z^2}$. The gas is in thermal equilibrium at a temperature T. Find the single particle partition function Z_1 for a trapped atom. (Hint : In spherical coordinates the volume element $dxdydz$ is replaced by $r^2 dr \sin\theta\, d\theta\, d\phi$. $\int_0^\infty x^2 e^{-x}\, dx = 2$). Find the entropy of the gas in terms of N, k and $Z_1(T, a)$. 4+3

10. The probability, ω(n), that an event charaterized by a probability p occurs n times in N trial, was shown to be given by the distribution

$$\omega(n) = \frac{N!}{n!(N-n)!} p^n \, (1-p)^{N-n}$$