Ref No. : EX/UG/SC/PHY/CORE-1/A/7/2019

B. Sc. (Physics) 1st Year 1st Sem. Exam. - 2019

Sub: Physics Time:2 Hours Full Marks : 50

Paper: CORE I

Use a separate answer script for each group.

Group: A

Answer any ***Five*** questions
(Each question carries equal marks)

1. (a) If F is constant force (independent of x, t and v) is acting on the particle of mass m, determine the velocity $v(t)$ and the trajectory $x(t)$ in terms of F, m, v_0, and x_0 where x_0 and v_0, are initial position and velocity at time $t = 0$. Sketch the graphs of $v(t)$ and $x(t)$ versus t.
(b) If F is the time-dependent force $F = A - Bt$, where A and B are positive constants, determine the velocity $v(t)$ and the trajectory $x(t)$ in terms of A, B, m, v_0 , and x_0 . The symbols have their usual meaning. [1.5+1.5+2]

2. A one-dimensional force $\vec{F} = -kx\hat{x}$, where k is a constant, acts on a particle of mass m.
(a) Calculate the potential energy $V(x)$ of the particle.
(b) Sketch the possible (for positive and negative values of k) graphs of $V(x)$. Use these graphs and conservation of energy to discuss the possible motions of the particle. [The second part will discuss the bounded and unbounded motion, identify classical turning points, classically forbidden regions, and points of stable and unstable equilibrium. [1+1+1+1+1]

3. A round, rigid object (such as a cylinder or sphere whose densities are axially or spherically symmetric, respectively) has mass M , radius a and moment of inertia I (about its axis or centre). The object is released from rest on a plane that is inclined at an angle α to the horizontal and for which the coefficient of static friction is μ_s. Suppose the object rolls without slipping. Determine the acceleration of the CM and the required frictional force in terms of M, g, a, α and I. Deduce that there is a critical angle of inclination [5]

$$\alpha_c = \tan^{-1}(1 + Ma^2/I)\,\mu_s.$$

Here, the critical angle of inclination α_c is the angle above which slipping occurs.

4. (a) State the Kepler's laws. Prove that the sector velocity $\frac{d\vec{A}}{dt}$ is constant. Here, the area dA sweeps out by the radius vector (joining the origin (sun) to the planet) in time dt. Further, show the conservation of angular momentum of the planet implies that the orbit lies in a plane.
(b) Calculate the radius of the circular orbit of a stationary Earth's satellite, which remains motionless with respect to its surface. The value of G is 6.67×10^{-11} in SI unit and mass of the Earth is $5.97 \times 10^{2}4$ kg. [1+2+1+1]

5. (a) In Newton's second law, if we set the force acting on a particle to zero, we obtain the acceleration of the object to be zero. Does it imply that Newton's first law of motion is included in the second law ?
(b) What are the limits of Newton's laws of motion. Discuss briefly the significance of the third law of motion. [1+3+1]

6. (a) State and derive the work-energy theorem. Does this theorem apply to all kinds of forces ?
(b) Forces for which $\oint \vec{F}.d\vec{r} \neq 0$ are called non-conservative forces. Show that the loss in the total energy over a path is equal to the work done by the non-conservative forces. [2+1+2]

7. (a) Calculate the angular velocity ($\vec{\omega}$) of the earth about its own axis. What is Coriolis force ? Calculate the strength of the Coriolis acceleration in the earth frame where velocity v (=50 m/s) of the particle and the angular velocity ($\vec{\omega}$) of the earth about its own axis are perpendicular.
(c) Explain with diagram "How do we observe two high tides and two low tides per day ?" In this view, we ignore the effect of the sun entirely. Further, we also assume that the oceans cover the whole surface of the globe. [0.5+1.5+3]

Group B

Answer any **Two** questions from **Q. No. 1** and **TWO** others from the rest.

1. (a) If $u = f(r)$ and $x = r\cos\theta$ and $y = r\sin\theta$ prove that $\left(2\frac{1}{2} \times 2 = 5\right)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(b) If z is a homogeneous function of degree n in x and y show that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

(c) Show that $\frac{\vec{r}}{r^3}$ is solenoidal.

2. (a) Find the conditions for a function $f(x, y)$ to be maximum or a minimum at a point (a,b).

(a) Show that the minimum value of $u = xy + a^3(\frac{1}{x} + \frac{1}{y})$ is $3a^2$. **(6+4)**

3. (a) Given $\emptyset = 2x^3y^2z^4$.

Show that $\nabla.\nabla\emptyset = \nabla^2\emptyset$ where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

(b) Verify Stoke's theorem for the function $\vec{F} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$ where closed curve is the unit circle in xy − plane bounding the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

(3+7)

4. (a) Determine the distance between two neighboring point in the curvilinear coordinate system. Hence write down the displacement vector in orthogonal curvilinear coordinate system.

(b) Find out the mathematical form of divergence of a vector ($\vec{V}$) in the curvilinear coordinate system using the definition of divergence.

(c) Hence find out the form of Laplacian (∇^2) operator in the Spherical polar coordinate system.

(2+5+3)

5. (a) Under what condition the solution of a linear second order differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$ is $y = (c_1 + c_2x)e^{mx}$? Prove it to support your answer.

(b) A body executes damped forced vibrations given by the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin\omega t$$

Solve it for both cases $\omega^2 \neq b^2 - k^2$ and when $\omega^2 = b^2 - k^2$. **(4+6)**