Ex./FM/5.6/44/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 1st Semester)

MATHEMATICS (HONOURS)

Unit - 5.6(c)

Mathematical Modelling - I

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.
(Symbols/Notations have their usual meanings)
Use a separate Answer-Script for each part.

PART - I (25 marks)

Answer ***Q. No. 5*** and any ***three*** from the rest.

1. The Jordan canonical form of the linear system is

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where α, β are positive real numbers. Discuss the stability of the origin (0,0) and draw the possible phase diagram of the trajectories. 8

2. What do you mean by local stability of an equilibrium point of the system $\dot{X} = F(X), X \in R^2$? Determine the nature of the equilibrium point and its stability for the autonomous system

(Turn over)

$$\dot{x} = \xi x - 2y,$$
$$\dot{y} = x + \xi y,\ \xi \in R,$$

with respect to ξ. 8

3. Stating the underlying assumptions, formulate the Kermack-McKendric epidemic model with temporary immunity. Find the endemic equilibrium point of the system and discuss its stability. 8

4. Describe an epidemic model with horizontal and vertical transmission. Find all equilibrium points of your model system and determine the disease eradication criteria. 8

5. Define prevalence of a disease. 1

PART - II (25 marks)

Answer ***Q. No.10*** and any ***three*** from the rest.

6. Derive the explicit solution of a single species logistic growth model of the form $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$ where r and K have their usual meaning. Give your comments on the result. 8

7. With suitable assumptions write down the classical Lotka-Volterra Prey-Predator model and discuss the dynamical behaviour of the model around the equilibrium points. 8

8. (a) Discuss the qualitative theory of single species continuous growth model of the form $\frac{dN}{dt} = f(N)$ where f(N) is a non-linear function. What do you mean by "basin of attraction".
   (b) Define functional response and numerical response of a prey-predator system. 5+3

9. Describe the two species mutualistic population growth model and discuss the qualitative behaviour of the system around the equilibrium points. 8

10. Draw the graph of Holling Type - I, II, III response functions. 1

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(Turn over)