5. (a) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	a_i
O_1	6	4	2	7	8
O_2	5	1	4	6	14
O_3	6	5	2	5	9
O_4	4	3	2	1	11
b_i	7	13	12	10	

(b) Consider the problem of assigning four operators to four machines. The asignment cost in rupees are given below. Operator 1 cannot be assigned to machine- III and operator 3 cannot be assigned to machine IV. Find the optimal cost of this assignment problem.

Machine

Operator	I	II	III	IV
1	5	5	–	2
2	7	4	2	3
3	9	3	5	–
4	7	2	6	7

8+8

6. Express (2,4,–3) as a linear combination of (1,3,1) and (0,2,5). 2

——— X ———

Ex:MATH/H/32/6.2/86/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

MATHEMATICS (HONOURS)

Linear Programming and Optimization

Paper : 6.2

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.
Symbols/Notations have their usual meaning.
Answer ***q.no. 6*** and any ***three*** from the rest.

1. (a) State and prove the fundamental theorem of linear programming problem.

(b) A company produces two types of leather belts, say type A and type B. Belt A is of a superior quality and belt B is of an inferior quality. Profits on the two types of belts are Rs.5 and Rs.4 per belt respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts are of type B, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires a fancy buckle and only 400 buckles are available for this per day. For belts of type B only 700 buckles are available per day. How should the company manufacutres the two types of belts in order to have a maximum over all profit.

2. (a) Show that although (2,3,2) if a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9,$$
$$3x_1 + 2x_2 + 5x_3 = 22,$$
$$x_1, x_2, x_3 \geq 0,$$

it is not a basic solution. How many basic solution of this system may have? Find all the basic feasible solutions of the given system of equations.

(b) Solve the following linear programming problem by simplex method :

Maximize $z = 3x_1 - x_2$

subject to

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$
$$x_1, x_2, x_3 \geq 0.$$

8+8

3. (a) Prove that the set of all feasible solutions to a linear programming problem is a convex set.

(b) Check whether the following set is convex or not.

$X = \{(x_1,x_2) \mid 2x_1 + x_2 \geq 20, x_1+2x_2 \leq 80, x_1+x_2 \leq 50, x_1, x_2 \geq 0\}$.

(c) Solve the following linear programming problem by using two phase simplex method.

Minimize $Z = x_1 - 2x_2 - 3x_3$

subject to

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0.$$

3+4+9

4. (a) Solve the following linear programming problem graphically :

Maximize $Z = 2x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 30$$
$$x_2 \geq 3$$
$$0 \leq x_2 \leq 12$$
$$0 \leq x_1 \leq 20$$
$$x_1 - x_2 \geq 0$$
$$x_1, x_2 \geq 0.$$

3+4+9

(b) Use duality to find the optimal solution of the following linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

subject to

$$x_1 + x_2 \geq 1$$
$$x_1 + x_2 \leq 7$$
$$x_1 + 2x_2 \leq 10$$
$$x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

8+8

(Turn Over)