6. (a) Construct a DFA that accepts the regular expression $10+(0+11)0^*1$.

   (b) Define a finite state machine (FSM). Design a FSM that adds two binary numbers of the form $x_1\ x_2 \ldots x_k$ and $y_1 y_2 \ldots y_k$. 7+(2+3)

7. (a) Prove that every regular language is a context free language, but its converse is not true.

   (b) Prove that the languages $L_1 = \{a^n\, b^n\, c^m \mid n \geq 0,\ m \geq 0\}$ and $L_2 = \{a^n\, b^m\, c^m \mid n \geq 0,\ m \geq 0\}$ on $\Sigma = \{a, b, c\}$ are context-free languages. Show that $L = L_1 \cap L_2$ is not a context-free language. 5+(4+3)

—— X ——

Ex:MATH/H/32/6.6/B/87/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

MATHEMATICS (HONOURS)

Discrete Mathematics - II

Paper : 6.6(b)

Time : Two hours | Full Marks : 50

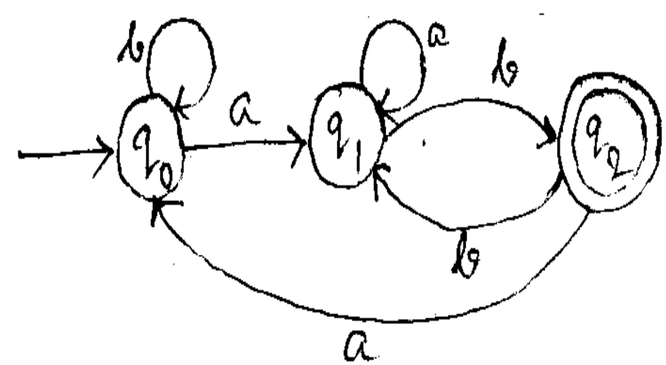
The figures in the margin indicate full marks.
Symbols/Notations have their usual meanings.

Answer ***q.no.1*** and any ***four*** from the rest.

1. State the pumping lemma for context free languages. 2

2. Define a deterministic finite automaton (DFA) M and the language L(M) accepted by the DFAM. State and prove the pumping lemma for regular languages. 2+2+(2+6)

3. (a) Let M be the DFA whose state transition diagram is shown in the following figure :



(Turn Over)

(i) Construct the transition table of M.

(ii) Which of the strings baba, baab, and abaab, are accepted by M ?

(iii) Find L(M).

(b) Show that the language $L = \{a^n b^n \mid n \geq 0\}$ over $\Sigma = \{a, b\}$ is not a regular language. (1+2+5)+4

4. (a) Let $M = (Q, \Sigma, q_0, \delta, F)$ be the non deterministic finite automaton (NDFA) such that $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$,

$F = \{q_2\}$, the initial state is q_0, and δ is given by

δ	a	b
q_0	$\{q_0, q_2\}$	$\{q_0\}$
q_1	ϕ	$\{q_0, q_2\}$
q_2	$\{q_1, q_2\}$	ϕ

(i) Draw the transition diagram of M.

(ii) Which of the strings aba, bbb, bba, and abb are accepted by M ?

(iii) Find L(M)

(iv) Find the corresponding DFA of M.

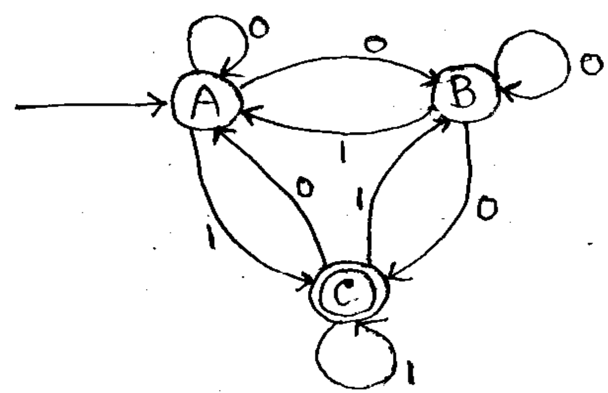
(b) Let M be a DFA and let G_M be its transition diagram. Then for any two states q_i and q_j, and a string $\omega \in \Sigma^*$, prove that $\delta^*(q_i, \omega) = q_j$, if and only if there is a directed walk P in G_M with the label ω from q_i to q_j where Σ is the input alphabet and δ^* is the extended transition function. (1+2+3+2)+4

5. (a) Let P and Q be two regular expressions over Σ. If P does not contain the empty word λ, then show that the equation in R :

$$R = Q + RP$$

has a unique solution. What is the unique solution ?

(b) Construct a regular expression recognized by the transition diagram of an NDFA given below :



(5+1)+6

(Turn Over)