Ex./FM/5.6B/43/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 1st Semester)

MATHEMATICS (Honours)

Discrete Mathematics - I

Paper - 5.6(b)

Time : Two hours Full Marks : 50

Use a separate answer-script for each part.
(Symbols have usual meanings, if not mentioned otherwise)

PART - I (25 marks)

Attempt the questions as follows.

1. Answer any ***two*** questions : 5x2=10

   (a) Prove that for any graph G with six points, G or $\overline{G}$ contains a triangle.

   (b) Prove that a graph is bipartite if and only if all its cycles are even.

   (c) Define n-cube. Find the number of points and lines in an n-cube.

2. Answer any ***three*** questions : 5x3=15

   (a) If every two points of a (p,q)-graph G are joined by a unique path, then prove that G is connected and $p = q + 1$.

(Turn over)

(b) Define the terms coboundary and cocycle. Prove that a cocycle is just a minimal nonzero coboundary.

(c) Prove that every hamiltonian graph is 2-connected and every nonhamiltonian 2-connected graph has theta subgraph.

(d) Define eulerian graph. If the set of lines of a connected graph G can be partitioned into cycles, then prove that G is eulerian.

(e) If G is a plane (p,q)-graph with no triangle, then prove that $q \le 2p - 4$. Hence show that $K_{3,3}$ is non-planar.

PART - II (25 marks)

Answr any ***five*** questions.

3. How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 0$ for which
   (i) each $x_i \ge 2$?
   (ii) where $x_1 \ge 3$, $x_2 \ge 2$, $x_3 \ge 4$, $x_4 \ge 6$ and $x_5 \ge 0$?

4. Prove that for any set A, $|P(A)| = 2^{|A|}$, using mathematical induction.

5. Prove the following identifies combinatorially :
   (i) ${}^{n}C_r \cdot {}^{r}C_k = {}^{n}C_k \; {}^{n-k}C_{r-k}$
   where $n \ge r \ge k \ge 0$.

   (ii) ${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$

6. Find how many integers between 1 and 1,000 are not divisible by 2, 3, 5 or 7.

7. Let D_n be the number of derangements of {1,2, ... n}. Find a general expression for D_n.

8. Prove the following identities
   (i) $F_0 + F_2 + F_4 + ... + F_{2n} = F_{2n+1}$
   (ii) $F_0 + F_1 + F_2 + ... + F_n = F_{n+2} - 1$
   when F_n represents the n^{th} Febonacci numbers.

9. If F_n satisfies the Fibonacci relation $F_n = F_{n-1} + F_{n-2}$ for $n \ge 2$, then there are constants C and C_2 such that

$$F_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

where the constants are completely determined by the initial conditions.

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