Ex:MATH/H/32/6.5/B/86/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

MATHEMATICS (HONOURS)

Differential Geometry

Paper : 6.5(b)

Time : Two hours — Full Marks : 50

(Symbols/Notation have their usual meaning)

Answer any ***five*** questions.

1. (a) Define equivalent paths. Prove that path equivalency is an equivalence relation. 1+4

   (b) Check whether the paths defined by

   $\vec{r}=(u, \sin u, e^u), -\infty<u<\infty$ and $\vec{r}=(\log v, \sin(\log v), v)$, $0 < v < \infty$ are equivalent or not. 2

   (c) Prove that $(\vec{a}\times\vec{b}).\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}=(\vec{a}.(\vec{b}\times\vec{c}))^2$. 3

2. Establish Serret-Frenet formulae for a space curve with their geometrical significance. 10

3. (a) Show that $\frac{d\vec{r}}{ds}\cdot\left(\frac{d^2\vec{r}}{ds^2}\times\frac{d^3\vec{r}}{ds^3}\right)=\frac{\tau}{\rho^2}$, where τ is the torsion and ρ is radius of curvature of the space curve $\vec{r}=r(s)$. 4

(Turn Over)

(b) Prove that a plane curve is entirely determined by the function k = k(s), apart from a rigid motion and the curve may be determined by quadrature. 6

4. (a) Define helix. Prove that for a helix the ratio of curvature and torsion is constant. 4

(b) Prove that for Bertrand mates the relation between curvature and torsion satisfy the relation $ak + b\tau = 1$, where a, b are constants. 6

5. (a) Find the parametric curves on the surface $\vec{r} = (u\cos v, u\sin v, bv)$, b being a constant. Also identify the surface. 4

(b) For the space curve x = 3 cost, y = 3 sint, z = 4t, find unit tangent vector, principal normal vector, binormal vector, curvature and torsion. Also determine the nature of the curve. 6

6. (a) For a surface $\vec{r} = (c\sin u\cos v, c\sin u\sin v, c\cos u)$ c being a constant, find the first fundamental form, angle between the parametric curves and unit normal vector $\vec{H}$. 5

(b) Also find the value of $d\vec{r}.d\vec{H}$ and explain its geometrical significance. 5

7. (a) If the first fundamental form of a surface is $ds^2 = (1 + u^2)\, du^2 + 2uv\, dudv + (1+v^2)dv^2$, prove that the angle between the parametric curves is

$$\tan^{-1}\frac{\left(1+u^2+v^2\right)^{1/2}}{uv}.$$ 4

(b) Find the area enclosed by the parametric curves on a surface $r = (u\cos v, u\sin v, 4u)$ for u = 2 to u = 3 and v = 0 to $v = \pi/6$. 6

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