8. Show that v(x,y) = – sin x sin hy is harmonic. Find the conjugate harmonic of v. (i.e., find an analytic function f = u + iv). 2+3

9. (a) Prove that for the transformation $\omega = \sin z$ the parallel lines x = constants and y = constants on z-plane transforms into confocal hyperbolas and confocal ellipses respectively on ω-plane.

(b) Find the bilinear transformation that maps the points z = 0, –i, 2i into the points ω = 5i, ∞, $-{}^{i}/_{3}$ respectively. 3+2

10. (a) Prove that circles are invariant under the transformation $\omega = \frac{1}{z}$.

(b) Determine the image of $|z-3| = 5$ under $\omega = \frac{1}{2}$. 3+2

11. If f(z) is a regular function, then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ 5

——— X ———

Ex:MATH/H/32/6.4/86/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

MATHEMATICS (HONOURS)

Analysis - IV

Paper : 6.4

Time : Two hours Full Marks : 50

Use a separate Answer-Script for each part.

PART - I (30 marks)

Answer any ***three*** questions.

1. (a) Let $f(x,y) = \frac{x^3}{x^2+y^2}$, $(x,y) \neq (0,0)$

$= 0$, $(x,y) = (0,0)$

Show that f is continuous at (0,0), all the directional derivatives exist at (0,0) but f is not differentiable at (0,0). 6

(b) If $f : R^n \to R$ is such that the partial derivatives exist and are bounded in a nbd. of $\tilde{a} \in R^n$ then show that f is continuous at $\tilde{a}$. 4

2. State and prove Implicit Function Theorem for two variables. 10

(Turn Over)

3. (a) Let $f : R^n \to R^m$ be differentiable at $\tilde{a} \in R^n$ and $g : R^m \to R^p$ be differentiable at $\tilde{b} = f(\tilde{a}) \in R^m$. Then prove that $h = g \circ f : R^n \to R^p$ is differentiable at $\tilde{a} \in R^n$. 7

(b) Let $f : R^2 \to R^2$ and $g : R^2 \to R$ be defined by $f(x,y) = (x^2, x^2+2y)$ and $g(x,y) = x + \cos y$. Calculate $h'(\tilde{a})$, where $h = g \circ f$ and $\tilde{a} = \left(1, -\frac{1}{2}\right)$. 3

4. (a) State and prove The Mean Value Theorem for a function from R^n to R^m. 5

(b) Let

$$f(x,y) = xy\frac{x^2 - y^2}{x^2 + y^2} \quad (x,y) \neq (0,0)$$
$$= \quad 0 \qquad (x,y) = (0,0)$$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. State a sufficient condition for the equality of f_{xy} and f_{yx} at a point (a,b). 4+1

5. (a) Let $f : \left(R^n \to R^n\right)$ be continuous on $\bar{B}$ where $B = B(\tilde{a}, r)$ and $\partial B = \bar{B} \setminus B$. Assume all the first order partial derivatives exist on B and $J_f(\tilde{x}) \neq 0 \quad \forall \tilde{x} \in B$. If $f(\tilde{x}) \neq f(\tilde{a}) \quad \forall \tilde{x} \in \partial B$ then show that $f(\tilde{a})$ is an interior point of f(B). 6

(b) Find the maxima and minima of the function

$$f(x,y) = x^3 + y^3 - 3x - 12y + 20.$$ 4

PART - II (20 marks)

Answer any ***four*** questions.

6. (a) If $f(z) = \frac{z}{|z|}$ continuous at origin, which is defined for $z \neq 0$ and $f(0) = 0$? Justify.

(b) Define analytic function at a point z_0, with an example. 3+2

7. Prove that if $f(z) = u(x,y) + i\,v(x,y)$ is differentiable at z then at this point the first order partial derivatives of u and v exist and satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

Is the converse true ? Justify. 4+1

(Turn Over)