Ex:MATH/H/32/6.1/86/2019

## BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

## MATHEMATICS (HONOURS)

## Probability Theory

## Paper : 6.1

Time : Two hours Full Marks : 50

Answer any ***five*** questions.

(All notations have their usual meaning)

1. (a) If $A_1, A_2, \ldots\ldots\ldots, A_n, \ldots\ldots.$ is a sequence of events in sample space S such that $A_1 \subseteq A_2 \subseteq \ldots\ldots\ldots \subseteq A_n \subseteq \ldots..$ then

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$$ 3

(b) Give axiomatic definition of probability. Use that definition to derive frequency definition of probability. 3

(c) From the numbers 1, 2, 3, .... (2n+1) three are chosen randomly. Find the probability that these are in arithmetic progression. 3

(d) If two events A and B are independent, show that A and $\vec{B}$ are also independent, where $\vec{B}$ denotes the compliment of the event B. 1

(Turn Over)

2. (a) A die is thrown $n$ times, find the probability of getting even number of sixes. 2

(b) Prove that in Bernoulli trials with probability of failure $q$, the probability of at most k successes is

$$\int_0^q x^{n-k-1}(1-x)^k\,dx \Big/ \int_0^1 x^{n-k-1}(1-x)^k\,dx \qquad 4$$

(c) Two socks are selected at random and removed in succession from the drawer containing five brown socks and three green socks. List the elements of the sample space, the corresponding values ω of the random variable W, where W is the number of brown socks selected. 3

(d) What is the probability that in a company of 500 people only one person will have birthday on New Years day ? (Assume that a year has 365 days). 1

3. (a) A player repeatdly throws a coin and scores one point for a head and two points for a tail. If $p_n$ denotes the probability of scoring n points, then show that $2p_n = p_{n-1} + p_{n-2}$. Hence deduce an expression for $p_n$, and find its limiting value as n tends to infinity. 4

(b) The random variable X is distributed uniformly over the interval (0 , 2). Find the distribution function of the largest root of the quadratic equation $t^2 + 2t - X = 0$. 3



(b) Let two random variables X and Y have the joint density function

$$f(x,y) = x+y, \quad 0 < x,y < 1$$
$$= 0 \qquad \text{elsewhere}$$

What is the covariance between X and Y ? 3

(c) A point a is fixed in the interval (0,1). A random variable X is uniformly distributed in that interval. Find the correlation coefficient between X and the distance Y between X and a. For what value of a, X and Y are uncorrelated ? 4

—— X ——

(b) Two coins are to be flipped. The first coin will land on head with probability 0.6, the second with probability 0.7. Assume that the results of the flip are independent, and let X equals the total number of heads that results. Find P(X=1) and E[X]. 3

(c) Show that moment generating function of a uniform distribution over an interval (–a,a) is sinh at/at. Hence find mean and variance of the distribution. 4

6. (a) The joint probability density function of two variates X, Y is given by

$$f(x,y) = k(3x+y), \quad 1 < x < 3, \ 0 < y < 2$$
$$= 0 \qquad \text{elsewhere}$$

Find the value of the constant k and calculate P(X+Y<2). Examine if X, Y are independent. 5

(b) Show that conditional distribution of X on the hypothesis that Y = y for bivariate normal distribution is a normal variate with mean $\mu_1 + \rho\dfrac{\sigma_1}{\sigma_2}\left(y-\mu_2\right)$ and variance $\sigma_1^2\left(1-\rho^2\right)$. 5

7. (a) Prove that the correlation coefficient ρ(X,Y) between two random variables X and Y satisfies the relation $-1 \le \rho \le 1$. 3



(c) If X is the number of heads obtained in four tosses of a balanced coin, find the probability distribution of Y = 1/(1+X) and Y = $(X-2)^2$. 3

4. (a) For the Poisson distribution with parameter μ, prove that

$$\mu_{k+1} = \mu\left(k\mu_{k-1} + \frac{d\mu_k}{d\mu}\right) \qquad 3$$

(b) Prove that the first absolute moment about any point is minimum when taken about the median. 4

(c) The probability density function of a continuous distribution is given by :

$f(x) = \dfrac{3}{4}x(2-x)$, 0 < x < 2, compute the mean, variance and the coefficient of skewness $\gamma_1$. 3

5. (a) Suppose it is known that the number of items produced in a factory during a week is a random variable with mean 50.

1. What can be said about the probability that this week's production will exceed 100 ?
2. If the variance of the week's production is equal to 25, then what can be said about the probability that this week's production will be between 25 and 75 ? 3

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