Ref. No. Ex/FM/5.1/39/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3^{rd} Year, 1^{st} Semester)

MATHEMATICS (Honours)

Paper – 5.1

(Numerical Methods)

Full Marks: 50 Time: Two Hours

The figures in the margin indicate full marks

Use a separate Answer script for each part

(Symbols have their usual meaning)

(Part – I)

(30 Marks)

Answer any ***THREE*** questions from the following:

1. (a) State and prove sufficient condition of convergence of fixed-point iteration method and hence show that the root obtained is unique.

   (b) Find the rate of convergence of the iteration scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}, \quad n = 0,1,2,\cdots$$

   (c) Determine the values of a, b, c such that the formula

$$\int_0^h f(x)dx = h[af(0) + bf\left(\frac{4h}{3}\right) + cf(h)]$$

   is exact for polynomial of as high order as possible. [(3+2)+2+3]

2. (a) Derive Hermite's interpolation formula.

   (b) Prove that $\mu\delta \equiv \frac{\nabla+\Delta}{2}$.

   (c) Determine the step size h and the number of points n to be used in the tabulation of $f(x) = \cos x$ in the interval $[1, 2]$ so that the error in quadratic interpolation will be less than or equal to 5×10^{-6}. [5+2+3]

3. (a) Establish Newton-Cotes' quadrature formula to evaluate $\int_a^b f(x)dx$ and hence derive Simpson's 1/3 rule.

(b) If α, β are the roots of $x^2 + ax + b = 0$, show that the iteration $x_{n+1} = -\left(\frac{ax_n+b}{x_n}\right)$ will converge to α when $|\alpha| > |\beta|$. [(4+3)+3]

4. (a) From the following table

x	-1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

find the value of $f'(3)$ using Richardson's extrapolation.

(b) Compute the relative error in computing $y = x^3 + 3x^2 - x$ for $x = \sqrt{2}$, taking $\sqrt{2} = 1.414$

(c) Distinguish between round-off and chopping errors with an example. [5+3+2]

Use a separate Answer script for each Group

Part II

(20 Marks)

Answer any TWO questions

Symbols and notations have their usual meaning

All questions are of equal marks

5. Describe Gauss Jacobi's method of solving a system of linear algebraic equations and prove its convergence. (10)

6. Obtain a quadratic polynomial approximation of the function $f(x) = x^3 + 5$ in $[0, 1]$ using least square approximation. (10)

7. Apply Modified Euler method to solve the following differential equation to find $y(0.1)$ correct up to two decimal places taking $h = .05$.

$$\frac{dy}{dx} = 5xy, \ 0 < x < 1, \ y(0) = 1. \qquad (10)$$