(ii) Show that $\rho = \frac{\left(k^1\right)^2}{h} + h$ 4

(notations are the usual ones)

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Ex:MATH/H/32/6.5/A/86/2019

BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 2nd Semester)

MATHEMATICS (HONOURS)

Mechanics - IV

Paper : 6.5(a)

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.
Notations and Symbols have their usual meaning.

1. Answer any ***five*** questions : 8x5=40

(a) A fine uniform circular tube is held in a vertical plane and contains two liquids of densities ρ and ρ^1 ($\rho > \rho^1$) which subtend angles 2θ and $2\theta^1$ at the centre. If the radius passing through the surface of separation makes an angle α with the vertical then show that

$$\rho \sin\theta \sin(\theta + \alpha) = \rho^1 \sin\theta^1 \sin(\theta^1 - \alpha)$$

(b) ABC is a triangle immersed vertically in water with C in the surface and the sides AC, BC equally inclined to the surface. Prove that the vertical through C divides the triangle into two others, the thrusts on which are in the ratio

$$b^3 + 3ab^2 : a^3 + 3a^2b$$

(c) A quadrilateral is immersed in water with two angular points in the surface and the other two at depths 'a' and 'b'. If α and β are the depths below the surface of its centre of gravity and centre of pressure respectively, show that

$$6\alpha\beta + ab = 3\alpha\,(a+b)$$

(d) A hemispherical bowl is filled with water and two vertical planes are drawn through its central radius, cutting off a semi-lune of the surface. If 2α be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is $\tan^{-1}\left(\sin\alpha/\alpha\right)$.

(e) A heavy uniform rod of small cross-section can turn freely in a vertical plane about a hinge placed at a quarter of the length of the rod from one extremity and at a height one-eighth of the length above the surface of a liquid. If d_1 and d_2 ($> d_1$) be the densities of the rod and the liquid, show that the rod rests at an inclination to the vertical equal to

$$\cos^{-1}\frac{1}{2}\sqrt{\frac{d_2}{9d_2-8d_1}}$$

(f) A closed circular cylinder is just filled with water, and rotates about its axis which is vertical. If the total thrust on the bottom is n times that on the top, then

the angular velocity is $\frac{2}{d}\sqrt{\frac{gh}{n-1}}$, where h is the height and d is the radius of the cylinder.

(g) A hollowcone whose weight is half that of the water it can contain, floats in water in stable equilibrium with its axis vertical and vertex downwards. Prove that if α be the semi-vertical angle of the cone,

$$\cos^6\alpha < \frac{729}{1024}$$

2. Answer any ***one*** question.

(a) (i) A given mass of liquid is in equilibrium under the action of a system of conservation forces. Write down the pressure equation at a point. Show that the surfaces of equi-pressure, equi-density and equi-density energy coincide. 6

(ii) If a given volume of liquid is at rest under forces whose components per unit mass are y(a–z), x(a–z), xy, then show that the density will be proportional to $\frac{1}{xy(a-z)}$. 4

(b) (i) Show that the tangent plane at any point on the surface of buoyancy is parallel to the corresponding position of the plane of floatation. 6