BACHELOR OF SCIENCE EXAMINATION, 2019

(3rd Year, 1st Semester)

MATHEMATICS (Honours)

Paper - 5.5A

(MECHANICS III)

Full Marks:50 Time: Two Hours

(Answer any five questions)

(Symbols/Notations have their usual meanings)

1. (a) If the moments and products of inertia of a plane lamina about two perpendicular axes in the plane are known, find the product of inertia about any perpendicular lines through their point of intersection. 4

(b) Find the moment of inertia of a right circular cone of mass "M" whose height is 4 unit and radius of whose base is 1 unit, about a line through the centre of gravity of the cone perpendicular to its axis. 4

(c) Show that the momental ellipsoid at the centre of an elliptic plate

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{is}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+z^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=constant.$$ 2

2. Find the condition for which a given straight line will be a principal axis of a material system at a particular point. Also determine the other two principal axes. Further prove that, if the given straight line passes through the centre of gravity of the material then it will be principal axis at any point of its length. 10

3. A uniform lamina is bounded by a parabolic arc, of latus rectum "4a", and a double ordinate at a distance "b" from the vertex. If $b = \frac{a}{3}(7 + 4\sqrt{7})$, show that two principal axes at the end of a latus rectum are tangent and normal there. 10

4(a) A bent lever whose arms are of length "a" and "b", the angle between them being 60^0,makes small oscillation in its own plane about the fulcrum, Find the length of the corresponding simple equivalent pendulum. 5

(b)Three equal particles are attached to a weightless rod equal distances "a" apart . The system is suspended from and is free to turn about a point of the rod at a distance x from the middle particle. Find the time of oscillation and show that it is least when $x = 0.82a$. 5

5(a) If a rigid-body rotates about a space-fixed axis, $\dot{\theta}$ be the angular velocity of the body about the axis at any instant and MK^2 the moment of inertia of the body about the axis, show that moment of momentum of the body about the fixed axis is $MK^2\dot{\theta}$. 4

(b) A uniform rod OA of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about the vertical line OB, and is inclined at a constant angle α to OB. Show that the value of α is $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. 6

6. A sphere of radius 'a' is projected up an inclined plane with velocity V and angular velocity Ω in the sense which would cause it to roll up; if $V>a\Omega$ and the coefficient of friction $> \frac{2}{7}\tan\alpha$, show that the sphere will cease to ascend at the end of time $\frac{5V+2a\Omega}{5g\sin\alpha}$ where 'α' is the inclination of the plane. 10

7(a) A heavy circular disc is revolving in a horizontal plane about its centre which is fixed. An insect of mass $\frac{1}{n}$th that of the disc, walks from the centre along a radius and then flies away. Show that the final angular velocity is $\frac{n}{n+2}$times the original angular velocity of the disc. 3

(b) If T be the kinetic energy and θ, φ be the generalized coordinates. Show that that Lagrange's equations are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = -\frac{\partial V}{\partial \theta} \quad \text{and} \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} = -\frac{\partial V}{\partial \varphi}$$

Where V is the potential function.