BACHELOR OF SCIENCE EXAMINATION, 2019
(3rd Year, 2nd Semester)
MATHEMATICS (HONOURS)
Mathematical Modelling - II
Paper : 6.6(c)

Time : Two hours   Full Marks : 50

The figures in the margin indicate full marks.
(Notations/Symbols have their usual meaning)

Answer any *five* questions.

1. Consider the single species fishery model with harvesting as :

   $\dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{k}\right) - h(t),$ and discuss the dynamic behaviour of the fish population described by the above model. What do you mean by biological over-exploitation ?   8+2

2. Explain the concepts of :

   (i)   Opportunity cost,

   (ii) Externality,

   (iii) Economic overfishing.   3+3+4

3. (a) What do you mean by interest and discount rates in resource management?

(b) Discuss the applicability of Cobb-Douglas production function in the fishery model of the form $\frac{dx}{dt} = F(x) - h(t), t \geq 0$.   5+5

4. (a) Define the fundamental genetic matrices of crossing by two genotypes.

(b) State and prove Hardy-Weinberg law.   4+6

5. (a) Define and discuss Newton's laws of cooling.

(b) Water is heated to the boiling point temperature 100°C. It is then removed from heat and kept in a room which is at constant temperature 60°C. After 3 minutes, the temperature of the water is 90°C. Find the temperature of water after 6 minutes. When will the temperature of water be 75°C and 61°C?   4+6

6. (a) Discuss the Richardson's model of two nations arms race.

(b) If the parameters of the Richardson's model are k = 3, l = 1, α = 4, β = 2, g = 6 and h = 1 then show that the defence levels of the two nations will be maintained at the armament level (3,2).   5+5

7. (a) What do you mean by a free terminal-value problem in optimal control?

(b) Find the optimal control which gives an extreme value of the functional

$$J = \int_0^1 u^2 \, dt,$$

where

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

and $x_1(0) = 1$, $x_1(1) = 0$, $x_2(0) = 1$ but $x_2(1)$ is free.

2+8

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