4. a) Let $\underset{\sim}{x} = (x_1, x_2 \ldots x_n)$ be a random sample of size n drawn from a population with pdf / pmf $f(x/\theta)$. Let $t_1(\underset{\sim}{x})$ & $t_2(\underset{\sim}{x})$ be two statistics.

   When do we say that $t_1(\underset{\sim}{x})$ is more efficient than $t_2(\underset{\sim}{x})$?

   b) If $\underset{\sim}{x}$ as above is a random sample from a population with mean μ, what condition must be imposed on the constants $a_1, a_2, \cdots, a_n$ so that $\sum_{1}^{n} a_i x_i$ is an unbiased estimator for μ? 4

5. a) What UMVUE ?

   b) State and prove Rao-Cramer Inequality. 3+7=10

6. In sampling from a Normal (μ, σ^2) population find the maximum likelihood estimates of μ and σ^2. 10

7. State and prove NP lemma. 10

Bachelor of Science Examination, 2019

(2nd Year, 1st Semester, Old)

Paper – 7 Stat

(Inference – I)

Time : Two hours　　　　Full Marks : 50

The figures in the margin indicate full marks.

(Symbols / Notations symbols have their usual meaning)

Attempt ***any five*** questions　　5×10=50

1. a) Define an unbiased estimator.

   b) Prove that sample mean is an unbiased estimator of the population mean. State the assumptions you use in the proof above. 3+7=10

2. a) Define a sufficient estimator with an example.

   b) State and prove Neyman's Factorisation theorem. 5+5=10

3. a) What is consistency ?

   b) Prove that if a population has finite variance, the sample mean is consistent. 3+7=10

[Turn over