Unit - 3 (15 marks)

Answer ***any three*** questions.

9. Apply the method of bisection to find the positive root of the equation $x^3 - x - 1 = 0$ correct upto two significant digits. 5

10. Find the positive root of the equation $3x - \cos x - 1 = 0$ by Newton-Raphson method correct upto three significant digits. 5

11. Solve the system of equations by Gauss elimination method :

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + 9z &= 16 \end{aligned}$$ 5

12. Solve the system of equations by Gauss-Jordan method :

$$\begin{aligned} 10x + y + z &= 12 \\ x + 10y + z &= 12 \\ x + y + 10z &= 12 \end{aligned}$$ 5

13. Use Simpson's one-third rule and Trapezoidal rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal sub-intervals, correct to three decimal places. 5

Ex/UG/GE-3/29/19

Inter B. Sc. Examination, 2019

(2nd year, 1st Semester)

GE-3 (Mathematics-II)

Mathematics-II

Time : Two hours Full Marks : 50

(Notations / symbols have their usual meaning)

Unit - 1 (15 marks)

Answer ***any three*** questions.

1. a) Find mod z and amp z (principal amplitude),

   where $z = 1 + \cos 2\theta + i \sin 2\theta \quad \frac{\pi}{2} < \theta < \pi$ 3

   b) Apply Descarte's rule of signs to find the nature of the roots of the equation $x^4 = 10$. 2

2. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $t^4 + t^2 + 1 = 0$ and n is a positive integer, then show that

$$\alpha^{2n} + \beta^{2n} + \gamma^{2n} + \delta^{2n} = 4\cos\left(\frac{2n\pi}{3}\right).$$ 5

3. Solve the equation by Ferrari's method

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$$ 5

[Turn over

4. a) The root 8 of the equation $x^3 + px^2 + qx + r = 0$ $(r \neq 0)$ are α, β, γ. Find the equation where roots are

$$\alpha - \frac{\beta\gamma}{\alpha},\ \beta - \frac{\gamma\alpha}{\beta},\ \gamma - \frac{\alpha\beta}{\gamma}. \qquad 2\tfrac{1}{2}$$

b) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, show that $A^2 - 2A + I_2 = O_2$.

Hence find $A^n, n \in N$. $2\frac{1}{2}$

5. Find the eigenvalues and eigenvectos of the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}. \qquad 5$$

Unit - 2 (20 marks)

Answer ***any two*** questions.

6. a) By eliminating the arbitry constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$, form a PDE.

b) Form a PDE by eliminating the arbitrary function f from

$$f(x+y+z,\ x^2+y^2+z^2) = 0.$$

c) Define with an exmaple the complete integral of a PDE.

d) Solve the PDE :

$$(x^2 - y^2 - z^2)p + 2xy\,q = 2xz. \qquad 2+3+2+3$$

7. a) Solve : $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial x \partial y} + 4\dfrac{\partial^2 z}{\partial y^2} = e^{2x-y}$.

b) By the method of separation of variables, solve the PDE :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

satisfying $u(x,0) = 6e^{-3x}$ 4+6

8. a) Solve the following ODE by reducing it to an exact differential equation :

$$(x^2 + y^2 + 2x)dx + 2y\,dy = 0.$$

b) Solve :

$$\frac{dx}{dt} = 4x - 2y,\ \frac{dy}{dt} = 5x + 2y. \qquad 4+6$$

[Turn over