Ex/INT/M/3.2/27/19(Old)

BACHELOR OF SCIENC EXAMINATION, 2019

(2nd Year, 1st Semester, Old Syllabus)

MATHEMATICS (HONOURS)

DIFFERENTIAL EQUATION - II

PAPER - 3.2

Time : Two hours | Full Marks : 50

Use a separate Answer-Script for each part

PART - I

(30 marks)

Answer ***any three*** questions

1. a) Solve the following initial value problem using power series method :

   $$y'' - xy = 0,$$

   where $y(1) = 2$ and $y'(1) = 0$. Write first four non-zero terms of the series. Find the radius of convergence and the interval of convergence of the series. 8

   b) Is it possible to find nontrivial solution of the first order ordinary differential equation $x^2 y' = y$, using power series method about $x = 0$? Justify your answer. 2

2. a) When can we apply Frobenius series method to find solution of an ordinary differential equation ? Explain with example. 2

[Turn over

b) Find Frobenius series solution about $x = 0$ of the differential equation

$$4xy'' + 2y' + y = 0.$$

Write the solution in terms of elementary functions. 8

3. a) Prove that eigenfunctions of a regular Strum-Liouville problem defined as

$$[p(x)v'(x)]' + q(x)v(x) + \lambda r(x)v(x) = 0, \quad a < x < b$$

$$\alpha v(a) + \beta v'(a) = 0$$

$$\gamma v(b) + \delta v'(b) = 0,$$

are real, where $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ and p(x), q(x), r(x) are real functions. Also prove that eigenfunctions corresponding to two district eigenvalues are orthogonal with respect to the inner product

$$< u, v >_r = \int_a^b u(x)v(x)r(x)dx.$$ 6

b) Find the eigenvalues λ_n and eigenfunctions $y_n(x)$ of the boundary value problem $y'' + \lambda y = 0, \ 0 < x < L$ together with boundary conditions $y(0) = y(L)$ and $y'(0) = y'(L)$. 4

b) State the convolution theorem for Laplace transform. Apply this theorem to evaluate

$$L^{-1}\left\{\frac{1}{(p+1)(p^2+1)}\right\}.$$ 5+5

8. a) Find $L^{-1}\left\{\log\frac{p^2+1}{p(p+1)}\right\}.$

b) Find $L\{t^{\alpha}\}$, where $t > 0$ and α being any real number > -1. Hence find $L\left\{t^{-1/2}\right\}.$

c) Find the solution of the following initial value problem :

$$y''(t) + a^2 y(t) = f(t)$$

$$y(0) = 1, \ y'(0) = 2.$$ 3+3+4

PART - II

(20 marks)

Answer ***any two*** questions

6. a) State and prove the existence theorem of Laplace transform.

   b) Solve the following simultaneous differential equations by the Laplace transform technique :

$$(D-2)x+3y=0$$

$$2x+(D-1)y=0,$$

where $D\equiv\frac{d}{dt}$, x and y are both functions of t, and

$x(0)=8,\ y(0)=3.$ 5+5

7. a) If $L\{f(t)\}=F(p)$, then show that

$$\int_0^\infty \frac{f(t)}{t}dt=\int_0^\infty F(x)dx.$$

Hence evaluate the integral

$$\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}dt.$$

4. a) Starting from the relation

$$e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)}=\sum_{n=-\infty}^{\infty}J_n(x)t^n,\ \text{prove that}$$

$$\cos(x\cos\theta)=J_0-2J_2\cos2\theta+2J_4\cos4\theta-\cdots$$

$$\sin(x\cos\theta)=2J_1\cos\theta-2J_3\cos3\theta+\cdots,$$

where $J_n(x)$ is the Bessel function of first kind of order n. 7

   b) Write first three terms of Bessel's function of first kind of order zero, $J_0(x)$. Hence find a rough estimate of the first positive zero of it. 3

5. a) Prove that

$$P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n,$$

where $P_n(x)$ is the Legendre polynomial of degree n. 7

   b) Find first three nonzero terms of the Fourier-Legendre series expansion of

$$f(x)=\begin{cases}0, & -1<x<0\\ 1, & 0\le x\le 1\end{cases}$$ 3