9. Derive Euler's equation for extreming the functional $I=\left[y(x)\right]=\int_{x_0}^{x_1}F(x,y,y1)\,dx$ with boundary conditions $y(x_0)=y_0,\ y(x_1)=y_1$. 8

——— X ———

Ex/MATH/H/22/4.2/2019(OLD)

BACHELOR OF SCIENCE EXAMINATION, 2019

(2nd Year, 2nd Semester, Old Syllabus)

MATHEMATICS (HONOURS)

Differential Equations - III

Paper : 4.2

Time : Two hours Full Marks : 50

Symbols and Notations have their usual meanings.
The figures in the margin indicate full marks.

PART - I (25 marks)

Answer any ***two*** questions from **q.no. 1–3** and any ***one*** question from the rest.

1. (a) Define a second order semilinear and quasilinear PDE in two independent variables x and y. Write down their general forms, with one example, in each case.

   (b) Form a PDE by eliminating the arbitrary constants a, b and c from Z = a(x+y) + b(x–y) + abt + c.

   (c) Solve the PDE

   $r - 4s + 4t = e^{2x+y}$ (2+2)+2+4

2. (a) Define complete integral and singular integral of a PDE.

(Turn Over)

(b) Find the complete integral and singular integral of the PDE

$$2xz - px^2 - 2qxy + pq = 0$$

(c) Solve the PDE

$$x_1 p_1^2 + x_2 p_2^2 - x_3 p_3^2 = 0$$

by Jacobi's method. 2+4+4

3. (a) Find the solution of a PDE of the form

$$f_1(x,p) = f_2(y, q)$$

Hence find the complete integral of the PDE

$$p^2q^2 + x^2y^2 = x^2q^2(x^2+y^2)$$

(b) Solve the PDE

$$\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$$ 7+3

4. (a) Classify the following PDEs :

(i) $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x\, u_{yy} - \cos x\, u_y = 0.$

(ii) $yu_{xx} + (x+y)\, u_{xy} + x u_{yy} = 0.$

(b) Form a PDE by eliminating the arbitrary function f from

$$f(x+y+z,\, x^2 + y^2 + z^2) = 0$$ 2+3

5. Solve the PDE : $(D + D')^2 z = e^{x-y}$ 5

PART - II (25 marks)

Answer ***q.no. 6*** and any ***two*** from the rest.

6. (a) State Jacabi equation for determining the extremals of the functional $I[y(x)] = \int_{x_0}^{x_1} F(x, y, y^1)dx$.

(b) Derive the transversality condition for the variational problem $I[y(x), Z(x)] = \int_{x_0}^{x_1} F(x, y, z, y^1, z^1)dx$, $y(x_0) = y_0$, $y(x_1) = y_1$, $z(x_0) = z_0$, $z(x_1) = z_1$, where the boundary point (x_0, y_0, z_0) is fixed while the other boundary point (x_1, y_1, z_1) moves along the curves $y = \phi(x)$, $z = \Psi(x)$. 1+8

7. Test for an extremum of the functional

$$I[y(x)] = \int_{\log\pi}^{\log a}\left(e^{-x}y^{1^2} - e^{x}y^2\right)dx \text{ with}$$

$$y(\log\pi) = y(\log a) = 0,\ a > \pi$$ 8

8. Find the curves on which the following functional attain extremum $I = \int_{-3}^{3} y^{1^2}dx, y(-3) = y(3) = 1,$ subject to the condition that the admissible curve cannot pass inside the area bounded by the circle $x^2 + y^2 = 5$. 8