BACHELOR OF SCIENCE EXAMINATION, 2019

(2nd Year, 2nd Semester, Old Syllabus)

MATHEMATICS (HONOURS)

Vector Calculus

Paper : 4.1

Time : Two hours — Full Marks : 50

Symbols and Notations have their usual meanings.

Answer any ***five*** questions.

1. (a) If $\bar{a}$ and $\bar{b}$ be two non-collinear vectors such that $\bar{a} = \bar{c} + \bar{d}$, where $\bar{c}$ is parallel to $\bar{b}$ and $\bar{d}$ is perpendicular to $\bar{b}$. Obtain expressions for $\bar{c}$ and $\bar{d}$ in terms $\bar{a}$ and $\bar{b}$.

   (b) Show that

$$\left(\bar{a}\times\bar{b}\right).\left(\bar{c}\times\bar{d}\right) = \begin{vmatrix} \bar{a}.\bar{c} & \bar{a}.\bar{d} \\ \bar{b}.\bar{c} & \bar{b}.\bar{d} \end{vmatrix}$$

   (c) Prove that

$$\left[\bar{p}\,\bar{q}\,\bar{r}\right]\left[\bar{a}\,\bar{b}\,\bar{c}\right] = \begin{vmatrix} \bar{p}.\bar{a} & \bar{p}.\bar{b} & \bar{p}.\bar{c} \\ \bar{q}.\bar{a} & \bar{q}.\bar{b} & \bar{q}.\bar{c} \\ \bar{r}.\bar{a} & \bar{r}.\bar{b} & \bar{r}.\bar{c} \end{vmatrix}$$

3+3+4

(Turn Over)

2. (a) Prove that

$$\bar{a}.\bar{\nabla}\left(\bar{b}.\bar{\nabla}\frac{1}{r}\right)=\frac{3(\bar{a}.\bar{r})(\bar{b}.\bar{r})}{r^5}-\frac{\bar{a}.\bar{b}}{r^3}$$ 5

(b) Prove that the vector

$$\bar{F}=\left(y^2\cos x+z^3\right)\hat{i}+(2y\sin x-4)\hat{j}+\left(3xz^2+2\right)\hat{k}.$$

is irrotational. Find the scalar potential ϕ for $\bar{F}$ such that $\bar{F}=\bar{\nabla}\phi$. 5

3. (a) Find the equations for tangent plane and normal line to the surface $z=x^2+y^2$ at the point (2,–1,5). 5

(b) If $u=3x^2y$ *and* $\upsilon=xz^2-2y$.

Evaluate $\bar{\nabla}\left(\bar{\nabla}u\,.\,\bar{\nabla}\upsilon\right)$. 5

4. Verify divergency theorem for $\bar{F}=2x^2\hat{i}-y^2\hat{j}+4xz\hat{k}$, taken over the region S in the first octant bounded by $y^2+z^2=9$ and $x=0$, $x=2$. 10

5. (a) State and prove Stoke's theorem. 7

(b) Show that Green's second identity can be written as

$$\int_v\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)d\upsilon=\int_s\left(\phi\frac{\partial\psi}{\partial\eta}-\psi\frac{\partial\phi}{\partial n}\right)ds$$ 3

6. (a) Show that under a rotation of rectangular axes the origin remaining the same, the vector differential operator $\bar{\nabla}$ remains invariant. 5

(b) Show that the necessary and sufficient condition that u(x,y,z), υ(x,y,z) and ω(x,y,z) be functionally related through the equation F(u,υ,ω) = 0, is $\left[\bar{\nabla}u\ \bar{\nabla}\upsilon\ \bar{\nabla}\omega\right]=0$. 5

7. (a) Show that the Frenet-Serret formulas can be written in the form $\frac{d\hat{t}}{ds}=\bar{\varpi}\times\bar{z}$, $\frac{d\bar{n}}{ds}=\bar{\varpi}\times\bar{n}$, $\frac{d\bar{b}}{ds}=\bar{\varpi}\times\bar{b}$.

Also determine $\bar{\varpi}$. 5

(b) If the n-th derivative of $\bar{r}$ *ω. r.* to S is given by

$$\bar{r}^n=a_n\bar{t}+b_n\bar{n}+c_n\bar{b}$$

Establish the following

$a_{n+1}=a'_n-k\,b_n$, $b_{n+1}=b'_n+k\,a_n-\tau c_n$,

$c_{n+1}=c'_n+\tau\,b_n$. 5

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