13. If $\phi(x) = f(x) + f(1-x)$, $x \in [0,1]$ and $f''(x) < 0$, for all $x \in [0,1]$. Show that $\varphi(x)$ is increasing on $\left[0, \frac{1}{2}\right]$ and decreasing on $\left[\frac{1}{2}, 1\right]$. 5

14. (a) If $f(x) = \tan x$, then show that

$$f^n(0) - {}^nc_2\, f^{n-2}(0) + {}^nc_4\, f^{n-4}(0) - \ldots.. = \sin n\pi/2$$

(b) If $\phi(x)$ is a polynomial in x and $\lambda$ is a real number, then prove that $\exists$ a root of $\phi'(x) + \lambda\varphi(x) = 0$, between any pair of roots of $\phi(x) = 0$. $2^1/_2+2^1/_2$

—— X ——

Ex./UG/M/CORE5/29/2019

**INTER BACHELOR OF SCIENCE EXAMINATION, 2019**

**(2nd Year, 1st Semester)**

**MATHEMATICS (HONOURS)**

**Theory of Real Functions**

**Paper : CORE - 5**

Time : Two hours Full Marks : 50

Answer any ***five*** questions.
Each question carry five marks

1. Let $f : R \to R$ be continuous on R and let $G = \{x \in R : f(x) > 0\}$, $F = \{x \in R : f(x) = 0\}$. Then show that G is open and F is closed in R.

2. Let $f, g : R \to R$ be continuous on R and D be a dense subset of R. If $f(x) = g(x)$ $\forall$ $x \in D$ then show that $f = g$ on R.

3. In the following either give an example of a continuous function f such that $f(S) = T$ or explain why there can be no such f
(i) $S = (0,1)$, $T = [0,1]$; (ii) $S = (0,1)$, $T = (1,2) \cup (2,3)$;

4. Prove that if a function $f : S \subset R \to R$ is uniformly continuous then for every pair of sequences $\{x_n\}$, $\{y_n\}$ in S with $\lim_{n\to\infty} (x_n - y_n) = 0$

(Turn over)

we have $\lim_{n\to\infty}(f(x_n) - f(y_n)) = 0$.
Hence show that $f : R \to R$ defined by $f(x) = x^2$, $\forall x \in R$ is not uniformly continuous on R.

5. Let $D_f$ denote the set of discontinuities of a monotone increasing or decreasing function defined on an interval I. Show that f can't have discontinuities of the 2nd kind and $D_f$ is countable.

6. Let $f : K \subset R \to R$ be a bijective continuous function. If K is compact then show that $f^{-1}$ is also continuous.

7. Let $f : [a,b] \to R$ be continuous on [a,b] such that $f(a)\, f(b) < 0$. Prove that there exists a point $c \in (a,b)$ such that $f(c) = 0$.

## GROUP - B

Answer any ***five*** questions.

8. State Rolle's theorem. What is its geometrical interpretation? Discuss applicability of Rolle's theorem to the function $f(x) = \cos\frac{1}{x}\ in\ [-1,1]$. 5



9. We have the Mean value theorem $f(x+h) = f(x) + h f'(x+\theta h)$, where $0 < \theta < 1$. Determine $\theta$ as a function of x and h where

   (i) $f(x) = x^2$ and (ii) $f(x) = e^x$. 5

10. Evaluate

    (i) $\lim_{x\to 0}\left(\frac{Sin\,x}{x}\right)^{\frac{1}{x^2}}$

    (ii) Find a and b such that

    $\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3} = 1$ 5

11. (a) Is mean value theorem applicable for the function $f(x) = 4 - (6-x)^{2/3}$ in [5,7].

    (b) Use mean value theorem of appropriate order to prove that $Sin\,x > x - \frac{x^3}{3!}$. 2+3

12. Suppose $f : [a,b] \to R$ be a differential function. Let $a < c < d < b$. If $f'(c) < 0$ and $f'(d) > 0$, then show that there exists a point $\xi \in (c, d)$ such that $f'(\xi) = 0$. 5

(Turn over)