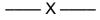
13. If $\phi(x) = f(x) + f(1-x)$, $x \in [0,1]$ and f''(x) < 0, for all $x \in [0,1]$. Show that $\phi(x)$ is increasing on $\left[0,\frac{1}{2}\right]$ and decreasing on $\left[\frac{1}{2},1\right]$.

14. (a) If $f(x) = \tan x$, then show that

$$f^{n}(0) - {n \choose c_2} f^{n-2}(0) + {n \choose c_4} f^{n-4}(0) - \dots = \sin n \frac{\pi}{2}$$

(b) If $\phi(x)$ is a polynomial in x and λ is a real number, then prove that \exists a root of $\phi'(x) + \lambda \phi(x) = 0$, between any pair of roots of $\phi(x) = 0$. $2^{1/2} + 2^{1/2}$



INTER BACHELOR OF SCIENCE EXAMINATION, 2019 (2nd Year, 1st Semester)

MATHEMATICS (HONOURS)

Theory of Real Functions
Paper: CORE - 5

Time: Two hours Full Marks: 50

Answer any *five* questions. Each question carry five marks

- 1. Let $f: R \rightarrow R$ be continuous on R and let $G = \{x \in R : f(x) > 0\}$, $F = \{x \in R : f(x) = 0\}$. Then show that G is open and F is closed in R.
- 2. Let f, g: R \rightarrow R be continuous on R and D be a dense subset of R. If $f(x) = g(x) \forall x \in D$ then show that f = g on R.
- 3. In the following either give an example of a continuous function f such that f(S) = T or explain why there can be no such f

(i)
$$S = (0,1), T = [0,1];$$
 (ii) $S = (0,1), T = (1,2) U(2,3);$

4. Prove that if a function $f: S \subset R \to R$ is uniformly continuous then for every pair of sequences $\{x_n\}, \{y_n\}$ in S with $\lim_{n\to\infty} (x_n - y_n) = 0$

(Turn over)

(3)

we have $\lim_{n\to\infty} (f(x_n) - f(y_n)) = 0$. Hence show that $f: R \to R$ defined by $f(x) = x^2$, $\forall x \in R$ is not uniformly continuous on R.

- 5. Let D_f denote the set of discontinuities of a monotone increasing or decreasing function defined on an interval I. Show that f can't have discontinuities of the 2nd kind and D_f is countable.
- 6. Let $f: K \subset R \to R$ be a bijective continuous function. If K is compact then show that f^{-1} is also continuous.
- 7. Let $f : [a,b] \to R$ be continuous on [a,b] such that f(a) f(b) < 0. Prove that there exists a point $c \in (a,b)$ such that f(c) = 0.

GROUP-B

Answer any *five* questions.

8. State Rolle's theorem. What is its geometrical interpretation? Discuss applicability of Rolle's

theorem to the function
$$f(x) = \cos \frac{1}{x}$$
 in [-1,1].

9. We have the Mean value theorem $f(x+h) = f(x) + h f'(x+\theta h)$, where $0 < \theta < 1$. Determine θ as a function of x and h where

(i)
$$f(x) = x^2$$
 and (ii) $f(x) = e^x$. 5

10. Evaluate

(i)
$$\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$

(ii) Find a and b such that

$$\lim_{x \to 0} \frac{a \sin 2x - b \sin x}{x^3} = 1$$

11. (a) Is mean value theorem applicable for the function $f(x) = 4 - (6 - x)^{\frac{2}{3}} \text{ in } [5,7].$

(b) Use mean value theorem of appropriate order to prove that $Sin x > x - \frac{x^3}{3!}$. 2+3

12. Suppose $f : [a,b] \to R$ be a differential function. Let a < c < d < b. If f'(c) < 0 and f'(d) > 0, then show that there exists a point $\xi \in (c,d)$ such that $f'(\xi) = 0$.

(Turn over)