B. Sc. Mathematics Examination, 2019

(2nd Year, 2nd Semester)

Mathematics

Statistics II

(GE - 4)

Time : Two hours Full Marks : 50

(25 marks for each part)

Use a separate Answer-Script for each part

PART - I

(Attempt question No. 1 and ***any three*** from the rest.)

1. State whether the following statement is true or false. $1 \times 7 = 7$

   a) BLUE stands for Best Linear unbiased Estimate.

   b) Cramer-Rao inequality gives an upper bound for the variance of the best estimator.

   c) UMVUE stands for Uniformly Maximum Variance Unbiased Estimator.

   d) If the population distribution is Normal (μ, 100) the following hypothesis is a composite one :

   $$H_O : \mu = O.$$

   e) Variance of a Chi-square random variable with 10 d.f. is 20.

f) In a likelihood function, the domain of definition is the Parametric spac, given a particular sample.

g) An unbiased estimator which is some function of a complete sufficient static is the best unbiased estimator.

2. Suppose that in our university $100 \times$ p% of students own a bicycle. ($p \in [0, 1]$) Based on a random sample of 100 students, find a maximum likelihood estimator of this unknown parameter p. 6

3. Define a Most Powerful test. State and Prove NP-lemma. 6

4. Explain briefly the basis of least square estimation, find a least square estimate of the population mean based on a random sample of size n from a normal $(\mu, 100)$ population. 6

5. Suppose a medicine manufacturer applies his new medicine on 20 patients to test $H_O : p = 90\%$ vs $H_1 : p = 60\%$ where p is the recovery rate of this new medicine. Suppose His test statistic is the following :

   Accept H_O if out of this 20 patients, at least 15 recoversand accept H_1 if this recovery number is less than equal to 14. Calculate α and β for this test.

6. Let $X_1, X_2, \cdots, X_n$ be Rectangular $(0, \theta)$

   a) Show that the sufficient statistic here is biased. Modify it to get an unbiased estimator of θ.

   b) Show that is also unbised.

   c) Test if any statistic among (a) and (b) above is complete sufficient.

PART - II

(Answer ***any five*** questions) 5×5

Each question carries 5 marks.

1. Define bias of an estimator. When is an estimator called unbiased ?

   If $x_1, x_2, \cdots, x_n$ are random observations on a Bernoulli variable X having the value 1 with probability θ and the value O with probability $(1-\theta)$, show that $\frac{\tau(\tau-1)}{n(n-1)}$ is an unbiased estimate of θ^2 where $\tau = \sum_{i=1}^{n} x_i$. 5

2. Define a consistant estimator with an example. If $\hat{\theta}_n$ is an unbiased estimate of θ_n with variance σ_n^2 and $\hat{\theta}_n \to \theta$ and $\sigma_n \to 0$ as then prove that $\hat{\theta}_n$ is a consistant estimate of θ. 5

3. Define a sufficient statistic with an illustration.

   Let $X_1, X_2, \cdots X_n$ be a random sample from a distribution with probability density function

   $$f(x,\theta) = e^{-(x-\theta)}, \theta < x < \alpha \; ; \; -\alpha < \theta < \alpha$$
   $$= 0, \text{ otherwise}$$

   Obtain sufficient statistic for θ. 5

4. Explain the term 'most efficient estimator' and define efficiency of any other estimator.

   If T_1 is a minimum variance unbiased (MVU) estimator for θ and T_2 is any other unbiased estimator of θ with efficiency e then prove that the correlation coefficient between T_1 and T_2 is given by $\rho = \sqrt{e}$. 5

5. What is Logistic curve in time series analysis ? Explain the method of three selected points applied to fit this curve to time series data regarding production in various years. 5

6. The data below given the average quarterly prices of a commodity for four years :

year	1st Question	2nd Question	3rd Question	4th Question
1980	40.3	44.8	46.0	48.0
1981	50.1	53.1	55.3	59.5
1982	47.2	50.1	52.1	55.2
1983	55.4	59.0	61.6	65.3

   Calculate the seasonal variation indices. 5

7. What is a index number ? Discuss its importance and uses.

   Explain i) time reversal test and

   ii) factor reversal test as applied to index numbers.

   Show that Fisher's ideal index number formula satisfies both these tests. 5