11. Find the radius of convergence of following power series :

i) $\sum_{n=1}^{\infty} n^3 x^3$ ii) $\sum_{n=0}^{\infty} \frac{2^n}{n^2} x^n$.

12. Let $f(x) = \sum_{n=0}^{\infty} c_n x^n, -1 < x < 1$. If $\sum_{n=0}^{\infty} c_n$ is convergent then show that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$. Give an example to show that $\lim_{x \to 1} f(x)$ may not exist if $\sum_{n=0}^{\infty} C_n$ is not convergent.

13. i) Find the Fourier series of the function $f(x) = \cos x + \sin 2x$ in $[-\pi, \pi]$.

ii) Give an example of a function f for which the Fourier series at a point may fail to coincide with the functional value at that point.

14. Use the Fourier series for $f(x) = |x|$ in $[-\pi, \pi]$ to find the sum of

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Ex/UG/Sc./Core/Math/08/2019

B.Sc. Mathematics Examination, 2019

(2nd Year, 2nd Semester)

Mathematics (Honours)

Core - 8

Riemann Integration & Series of Functions

Time : Two hours Full Marks : 50

PART - I

Symbols / Notations have their usual meanings.

(Answer ***any four*** questions)

1. Why do you assume that a function $f : [a, b] \to \mathbb{R}$ is bounded for its Riemann integrability on [a, b] ? If P_1 , P_2 are any two partitions of [a, b], then show that

$$S_*(f, P_1) \le S^*(f, P_2).$$ 5

2. Suppose a function f be bounded and integrable on [a, b], then prove that the function F defined as

$F(x) = \int_a^x f(t)dt, \; a \le x \le b,$ is continuous on [a, b]. Further if f is continuous at any print c in [a, b], then prove that F is derivable at C and $F'(c) = f(c)$. 5

3. What is an improper integral ? Test the convergence of

i) $\int_0^1 x^{n-1} \log x dx$

ii) $\int_0^1 x^{-3/2} \operatorname{Sin}\frac{1}{x} dx$ 5

4. If f be bounded and Riemann Integrable in [a, b], then prove that | f | is also bounded and Riemann integrable in [a, b] and

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$$

Is the converse of the above statement true ? Justify your answer. 5

5. Prove that $B(m,n)=\frac{\overline{|(m)}\cdot\overline{|(n)}}{\Gamma(m+n)}$, $m>0,\ n>0$ and hence evaluate $\overline{\left|\left(\frac{1}{2}\right)\right.}$. 5

6. If the function $f:[0,1]\to\mathbb{R}$ be defined by $f(x)=a^n$, $a^{n+1}<x\le a^n$, for $n=0, 1, 2, \ldots$ and $f(0)=0$,

where $0<a<1$

Is f Riemann integrable on [0, 1] ? 5

PART - II

(Answer ***any six*** questions)

(Each question carry five marks) 5×6=30

7. Let $\{f_n\}$ be a sequence of functions, continuous on a compact set E such that $f_n \to f$ pointwise on E. If $f_n(x) \ge f_{n+1}(x)$ for n = 1, 2, ... and for every $x\in E$ then show that $f_n \to f$ uniformly.

8. State Weierstrass's M-test for the uniform convergence of series of functions and use it to show that the series $\sum_{n=1}^{\infty}\frac{1}{n^2+x^2}$ is uniforly convergent.

9. Let $f:[0,1]\to R$ be a continuous function such that

$$\int_a^b x^n f(x)dx = 0,\ \ \forall n = 0,1,2,3\cdots$$

Then show that $f=0$.

10. Let $\{f_n\}$ be a sequence of functions on $[0,\infty)$ defined as follows :

$$f_n(x)=\frac{1}{n}\exp^{-nx}.$$

Then show that $\{f_n\}$ converges uniformly on $[0,\infty)$.