INTER B. SC. EXAMINATION, 2019

(1st Semester, Old)

MATHEMATICS

ANALYSIS I

UNIT : 3.3

Time : Two hours | Full Marks : 50

(Symbols and Notations have their usual meanings)

(Answer any five questions)

1. Suppose R be an Archimedean ordered field. Show that the following are equivalent :
   (i) Every non-empty bounded above subset of R has a least upper bound.
   (ii) For every sequence $\{I_n\}$ of nested closed intervals, $I_n = [a_n, b_n]$ with $inf\{b_n - a_n\} = 0$ the intersection $\cap_{n=1}^{\infty} I_n$ consists of exactly one point. **5+5**

2. Prove that in R
   (i) Finite intersection of open sets is open.
   (ii) Arbitrary union of open sets is open.
   (iii) Give examples to show that arbitrary intersection of open sets may not be open. **4+4+2**

3. (i) Show that a subset of R is compact if and only if it is closed and bounded.
   (ii) Justify whether $[-1, 2)$ and N are compact or not. **8+2**

4. (i) Let $A \subset R$. Show that a point $x \in A'$ (A' is the derived set of A) iff there exists a sequence $\{x_n\}$ of distinct points of A such that $x_n \longrightarrow x$ as $n \longrightarrow \infty$.
   (ii) Let f and g be continuous from R to R such that $f(x) = g(x)$ $\forall x \in D$ where D is a dense subset of R. Does it imply $f = g$ on R ? **6+4**

5. (i) Let $f : R \longrightarrow R$ be continuous on R and $f(x_0) > c$ for some x_0 in R. Then show that there exists a nbd. U of x_0 such that $f(x) > c$ $\forall x \in U$.
   (ii) In the following either give an example of a continuous function f such that $f(S) = T$ or explain whether there can be no such f :
   (1) $S = [-1, 1], T = [3, 7]$; (2) $S = [0, 1], T = (0, 1]$. **4+6.**

6. (i) Let $f : S \subset R \longrightarrow R$ be uniformly continuous. If $\{x_n\} \subset S$ is a Cauchy sequence then show that $\{f(x_n)\}$ is also a Cauchy sequence in R.

[Turn over

(ii) Show that the function $f(x) = 1/x$ and $f(x) = \sin 1/x$ are not uniformly continuous on $(0, \infty)$. **4+6.**

7. (i) Establish the convergence or divergence of the series whose n-th term is

$$(1)\ \frac{1}{(n(n+2))^{1/2}},\quad (2)\ \frac{3^n}{e^n}$$

(ii) Consider the alternating series

$$1 - 1/2 + 1/3 - \ldots$$

Show that the series is conditionally convergent. **6+4**