Ex/UG/Sc./Core/Math/TH/09/2019

Bachelor of Science Examination, 2019

(2nd Year, 2nd Semester)

Mathematics (Honours)

Paper : Core - 9

Multivariate Calculus

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.

(Symbols / Notations have their usual meanings)

Use a separate Answer-Script for each part

PART - I

(25 Marks)

(Answer ***any five*** questions)

1. Show that the limit of the function f(x, y) exists at the origin, but the repeated limits do not exist, where

$$f(x,y)=\begin{cases} x\sin\frac{1}{y}+y\sin\frac{1}{x}, & \text{for} \quad (x,y)\neq(0,0) \\ 0 & \text{for} \quad (x,y)=(0,0) \end{cases}$$

5

2. Show that the function

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{when} \quad (x,y)\neq(0,0) \\ 0 & \text{when} \quad (x,y)=(0,0) \end{cases}$$

is continuous at (0, 0). 5

3. If the function f(x,y) is differentiable at (a, b), then show that both the partial derivatives f_x and f_y exist at (a, b). Further show that the total differential of the function f(x,y) is given by

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy.$$ 5

4. Given that $f'(x)=\dfrac{1}{1+x^2}$ and f(0) = 0. By the help of Jacobian, show that

$$f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right).$$ 5

5. Using Lagrange's multiplier method, show that the largest rectangle with a given perimeter is a square. 5

6. If $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vectors, then show that any vector $\vec{r}$ can be expressed as

the transformation x = (u+v)/4 and y = (v – 3u) / 4. 5

13. Evaluate the line integral

$$\int_C xdx+(x^3+3xy^2)dy$$

(a) directly (b) using Green's Theorem, where C is the closed curve consisting of the line from (–2, 0) to (2, 0) and the arc of the semicircle $y=\sqrt{4-x^2}$ from (2, 0) to (–2, 0). 5

14. Find the value of surface integral

$$\iint_S A.n\,dS,$$

where S is the closed surface consisting of the planes z = 0, z = 3 and the surface of the cylinder $x^2+y^2=4$. 5

PART - II

(25 Marks)

(Symbols / Notations have their usual meanings.)

(Attempt ***any five*** questions)

8. Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$. 5

9. Evaluate

$$\iint_D y^3 dA,$$

where D is the triangular region with vertices (1, 1), (0, 2) and (3, 2). 5

10. Change the order of integration and hence evaluate

$$\int_0^3 \int_0^{\sqrt{9-y}} dx dy.$$ 5

11. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Find its centre of mass if the density at any point is proportional to the square of its distance from the origin. 5

12. Find the image of the parallelogram R with vertices at $A(-1, 3)$, $B(1, -3)$, $C(3, -1)$ and $D(1, 5)$ in $xy-$plane under

$$\vec{r} = \frac{[\vec{r}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{a} + \frac{[\vec{a}\ \vec{r}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{b} + \frac{[\vec{a}\ \vec{b}\ \vec{r}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{c}.$$ 5

7. If ϕ and ψ are scalar functions of x, y and z possessing first and second order partial derivatives, then prove that

$$\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi \cdot \nabla\psi + \psi\nabla^2\phi.$$ 5