Ref. No.: Ex/INT/M/8S/27/19(OLD)

Inter B.Sc 1st Sem Exam, 2019(old)

Mathematics(Subsidiary)

Methods of Series Solution of ODE & Special Functions

(Paper-8S)

Full Marks:50 Time: Two Hours

(Notations and symbols have their usual meanings.)
Answer any five questions.

1. Find power series solution in powers of $(x-1)$ of the initial value problem: (10)

$$\begin{aligned} x\frac{d^2y}{dx^2}+\frac{dy}{dx}+2y &= 0 \\ y(1)=2, \quad y'(1) &= 4 \end{aligned}$$

Write first six nonzero terms in the series.

2. Find Frobenius series solution about the origin of the differential equation: (10)

$$2x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+(1-x^2)y = 0.$$

Write at least first three nonzero terms in each series.

3. (a) Prove that (7)

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & ,m \neq n \\ \frac{2}{2n+1} & ,m = n \end{cases}$$

where $P_n(x)$ is the Legendra polynomial of degree n.

(b) Use generating function of Legendra polynomials to prove (3)

i. $P_n(1) = 1$

ii. $P_{2n}(0) = (-1)^n \frac{1.3.5.........(2n-1)}{2^n n!}$

4. (a) Starting from the relation (7)

$$e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} J_n(x)t^n, \qquad \text{prove that}$$

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)d\theta,$$

where $J_n(x)$ is the Bessel function of first kind of order n.

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(b) Write first three terms of Bessel's function of first kind of order zero. Hence find a rough estimate of the first positive zero of it. (3)

5. State the orthogonality property of Chebyshev ploynomials of first kind. Draw graph of first five Tchebyshev polynomials of first kind. Find the Tchebyshev series expansion of $\sin(\cos^{-1} x)$. Write first five terms of the series. (10)

6. (a) Prove the Rodrigue's formula for Hermite polynomials. (4)

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$$

(b) Starting from generating function of Hermite polynomials, prove that

i. $\frac{dH_n}{dx}(x) = 2nH_{n-1}(x)$ (2)

ii. $\frac{d^m}{dx^m}[H_n(x)] = \frac{2^m n!}{(n-m)!} H_{n-m}(x), m < n$ (2)

iii. $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$ (2)

7. (a) Starting from Rodrigue's formula, prove that Legendre polynomial of degree n can be expressed as (4)

$$P_n(x) = \frac{1}{2^n} \sum_{r=0}^{N} \frac{(-1)^r (2n-2r)! x^{n-2r}}{r!(n-r)!(n-2r)!}.$$

(b) Prove that (4)

$$\int J_5(x)dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x) + c,$$

where $J_n(x)$ is Bessel's function of first kind of degree n.

(c) Write generating function of Tchebyshev polynomials. (2)