Ref. No. : Int/M/3.1/27/19 (OLD)

INTER B. SC. EXAMINATION, 2019 (OLD)

(1st Semester)

MATHEMATICS (Honours)

Paper - 3.1

(Mechanics - II)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer **Q. No. 10** and **any six** questions from the rest.

1. A particle whose mass increases through condensation of moisture at a constant rate $\frac{m_0}{\tau}$, where m_0 is the initial mass of the particle and τ is a constant, moves freely under gravity. The particle is projected from the origin of a rectangular coordinate system whose x-axis is horizontal and y-axis is vertically upwards. Prove that the coordinates of the particle when its mass is m are given by

$$x = u\tau \log\left(\frac{m}{m_0}\right),$$
$$y = \frac{1}{4}g\tau^2\left\{1-\left(\frac{m}{m_0}\right)^2\right\} + \left(v\tau + \frac{1}{2}g\tau^2\right)\log\left(\frac{m}{m_0}\right),$$

where u, v are the components of velocities of the ball at the origin along x-axis and y-axis respectively. Show also that if $v > 0$, the greatest height attained by the particle is

$$\frac{1}{2}\left(v\tau + \frac{1}{2}g\tau^2\right)\log\left(1+\frac{2v}{g\tau}\right) - \frac{1}{2}v\tau$$

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2. If the acceleration of a particle describing a plane curve is resolved into two components, one parallel to the initial line and the other along the radius vector, prove that the components are

$$-\frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) \quad\text{and}\quad \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 + \frac{\cot\theta}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right),$$

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where (r, θ) is the polar coordinate of the particle at any instant t.

8

3. A particle describes a rectangular hyperbola $x^2 - y^2 = a^2$, under a force directed from the centre. Find the law of force. Show that the angle θ described about the centre in time t after leaving the vertex is given by the equation

$$\tan\theta = \tanh(\sqrt{\mu}t) ,$$

where μ is the force per unit mass at a distance unity from the centre.

8

4. A particle of mass m is moving under the action of central force mF in a medium which exerts a resistance equal to $k \times (velocity)^2$ per unit mass. Write the equation of motion of the particle along radial and cross-radial directions. Hence show that the differential equation of the path of the particle is

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h_0^2u^2}e^{2ks} ,$$

where s is the length of the arc described in time t and h_0 is the initial moment of momentum per unit mass about the centre of force.

8

5. A particle is moving under the action of central force $\mu\left\{2\left(a^2+b^2\right)u^5 - 3a^2b^2u^7\right\}$ per unit mass. If it is projected from an apse at a distance a with a velocity $\frac{\sqrt{\mu}}{a}$, prove that the polar equation of its path is $r^2 = a^2\cos^2\theta + b^2\sin^2\theta$.

8

6. A smooth parabolic tube is placed in a vertical plane with vertex downwards. A particle slides down the tube from rest under the influence of gravity. Prove that in any position the reaction of the tube on the particle is $2w\frac{h+a}{\rho}$, where w is the weight of the particle, ρ is the radius of curveture, $4a$ is the latus rectum and h is the original height of the particle above the vertex.

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7. Let a particle of mass m be acted upon by equal constant forces mf tangentially and normally to the path of the particle. If v is the magnitude of the resultant velocity of the

particle at any time t and $\frac{fv^2}{k^2}$ is the resistance of the medium per unit mass, prove that the intrinsic equation of the path of the particle is

$$k^2(e^{2fs/k^2} - 1) = u^2(e^{2\psi} - 1) \ ,$$

where u is the velocity of projection.

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8. Find the expressions of velocities and accelerations of a particle moving in a plane with respect to a set of mutually perpendicular rotating axes.

8

9. (a) A particle of mass m describes an ellipse under the action of the central force $\frac{m\mu}{r^2}$ with a focus as the centre of force. Prove that the velocity at the end of the minor axis is the geometric mean of the velocities at the ends of any diameter.

4

(b) The pedal equation of nearly circular orbit under a central force is $p^2(a^{m-2} - r^{m-2}) = b^m$, $m > 2$. Find the law of force. Show that the orbit is stable and the apsidal angle is $\frac{\pi}{\sqrt{m}}$.

4

10. Consider a particle of mass m moving in a plane curve under the action of any force. Prove that the resultant velocity of the particle is always acting along the tangent to the path of the particle.

2