Ex/MATH/S/22/12.5/2019(OLD)

BACHELOR OF SCIENCE EXAMINATION, 2019

(2nd Year, 2nd Semester, Old Syllabus)

MATHEMATICS (SUBSIDIARY)

Paper : 12.S

Time : Two hours | Full Marks : 50

Figures in the margin indicate full marks.
Symbols and Notations have their usual meanings.

PART - I (30 marks)

Answer any ***two*** questions.

1. (a) Form a PDE by eliminating the arbitrary constants a, b and c from

$$(x-a)^2 + (y-b)^2 + z^2 = c^2$$

(b) Let z be a function of two independent variables x and y; and u, v are two given functions of x, y and z connected by the relation $f(u,v)=0$. where f is arbitrary. By eliminating the arbitrary function f, form a PDE.

(c) By eliminating the arbitrary function f and g form a PDE from

$$z = yf(x) + x\,g(y).$$

(d) Solve the PDE :

$y^2p - xyq = x(z-2y)$ 3+4+4+4

(Turn Over)

2. (a) Solve the PDE :

$$(y+z)p-(z+x)q=x-y$$

(b) Solve : $(D^2-DD')z=\cos x\cos 2y$.

(c) Find the half range cosine series expansion for the function $f(x)=x^2$ in the range $0\le x\le\pi$. 4+5+6

3. (a) Obtain a Fourier series representation for the function $f(x)=|x|$ in $-\pi<x<\pi$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$$

(b) Solve : $\frac{\partial^2 z}{\partial x^2}=z=0$, given that when $x=0$, $z=e^y$ and $\frac{\partial z}{\partial x}=1$. 10+5

PART - II (20 marks)

Answer any ***two*** questions.

4. (a) Show that $L\{e^{-at}f(t)\}=F(p+a)$, where $L\{f(t);p\}=F(p)$.

Hence find $L\left\{e^{-3t}(2\cos 5t-3\sin 5t)\right\}$.

(b) If $L\{f(t);p\}=F(p)$, then show that

$$L\{f(at);p\}=\frac{1}{a}F(p/a).$$

Hence find $L\left\{\frac{\sin at}{t}\right\}$, given that $L\left\{\frac{\sin t}{t};p\right\},=\tan^{-1}\left(\frac{1}{p}\right)$. 5+5

5. (a) Show that $L\int_0^t f(t)dt=\frac{F(p)}{p}$, given that $L\{ft);p\}=F(p)$. Hence evaluate $L\int_0^t e^{-t}\cos t\,dt$.

(b) Evaluate : $L^{-1}\left\{\log\frac{p^2+1}{p(p+1)}\right\}$ 5+5

6. (a) Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{p(p^2+4)}\right\}$.

(b) Solve : $y''(t)+y(t)=t$, $y(0)=1$, $y'(0)=-2$. 4+6

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