7. a) Verify stokes theorem for

$\overline{F} = y\hat{i} + z\hat{i} + x\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and π is its boundary. 6

b) By using stoke's theorem

evaluate $\int_{\mu} (e^x dx + 2y dy - dz)$

where μ is the curve $x^2 + y^2 = 4, \ z = 2$ 4

Ex/INT/M/7S/27/2019(Old)

Inter B. Sc. Examination, 2019

(1st Semester, Old)

Mathematics

Paper - 7S (Vector Calculus)

Time : Two hours Full Marks : 50

Answer ***any five*** questions

(Notation symbols have their usual meaning)

1. a) Prove that a necessary and sufficient condition that a. vector function $\overline{u}$ has constant length is $\overline{u}.\frac{d\overline{u}}{dt} = 0$

b) Given that $\overline{r} = a\cos t\ \hat{i} + a\sin t\ \hat{j} + bt\ \hat{k}$.

show that

(i) $\left(\overline{r'}\right)^2 = a^2 + b^2$

(ii) $\left(\overline{r'} \times \overline{r''}\right)^2 = a^2\left(a^2 + b^2\right)$

Where $\overline{r'} = \frac{d\overline{r}}{dt}$ 2+4

[Turn over

2. a) Find the directional derivative of $f(x,y,z) = xy + yz + zx$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at $(1, 2, 0)$ 5

b) Find grad $((\bar{r})^m)$ where $(\bar{r})^2 = x^2 + y^2 + z^2$ 5

3. a) If $\bar{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

then find div $\bar{F}$ and curl $\bar{F}$ 3

b) Determine the constant a so that the vector

$$\bar{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$$

becomes solenoidal. 3

c) If $\bar{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

and $\bar{C}$ is any constant vector

$\bar{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$ then

then show that

$\text{div}(\bar{r} \times \bar{C}) = 0,\ \text{Curl}(\bar{r} \times \bar{C}) = -2\bar{C},$ 4

4. a) Prove that

$$\nabla \times (\nabla \times \bar{F}) = \nabla(\nabla \cdot \bar{F}) - F \cdot (\nabla^2 F)$$

where $\bar{F}$ is a vector function. 6

b) If $\bar{A} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$,

find $\nabla \times (\nabla \times A)$ 4

5. a) Consider the vector function

$$\bar{F} = (x^2 + y^2)i - 2xy\ j$$

and C is the boundary of the rectangle obtained by

$y = 0, x = a, y = b, x = 0.$

Prove that $\int_C \bar{F} \cdot d\bar{r} = -2ab^2$ 5

b) Evaluate $\int_S \bar{F} \cdot d\bar{s}$ for the vector field

$$\bar{F} = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$$

where s is the surface of the cube bounded by

$x = 0, y = 0, z = 0, x = 1, y = 1, z = 1.$ 5

6. a) State Gauss Divergence theorem. Express the theorem in Cartesian form. 4

b) Verify Gauss Divergence theorem for the vector function.

$$\bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelopiped

$0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$ 6