PART - III (15 Marks)

Answer ***any three*** questions.

9. State and prove the first Mean Value theorem of Integral calculus. 5

10. Test the convergence of the following :

i) $\displaystyle\int_0^{\infty}\frac{dx}{x^2+2x+2}$,

ii) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}dx$. 3+2

11. a) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

b) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}}\sin^5 x dx$. 2+3

12. Evaluate $\iint (x^2+y^2)dxdy$ over the region enclosed by the triangle having its vertices at (0, 0), (1, 0) and (1, 1). 5

13. Evaluate $\displaystyle\iint\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)dxdy$ over the positive quadrant of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. 5

Ex/UG/Sc./GE/Math/02/2019

Bachelor of Science Examination, 2019

(2nd Year, 2nd Semester)

MATHEMATICS

Mathematics - I

GE - 2

Time : Two hours Full Marks : 50

Use a separate Answer-Script for each part

PART - I (15 Marks)

Symbols and notations have their usual meanings.

Answer ***any three*** questions.

1. State Lagrange's Mean value theorem. What is it's geometrical interpretation ? Using this theorem, prove that

$$\frac{x}{1+x}<\log(1+x)<x, \text{ for all } x>0$$ 5

2. a) Examine the continuity of the function

$$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & \text{when } x\neq y\\ 0, & \text{when } x=y,\end{cases}$$

at (0, 0).

b) If $y = x^{n-1} \log x$, then prove that

$$y_n = \frac{(x-1)!}{x}$$ $2\frac{1}{2}+2\frac{1}{2}$

3. State Euler's theorem for homogeneous function of two variables.

If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, then show that

$$x^2 \frac{\partial u^2}{\partial x^2} + 2xy \frac{\partial u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial u^2} = (1 - 4\sin^2 u). \quad \sin 2u.$$

1+4

4. Let

$$f(x,y) = \begin{cases} x \sin\frac{1}{y} + y \sin\frac{1}{x}, & \text{where} \quad xy \neq 0 \\ 0, & \text{when} \quad xy = 0 \end{cases}$$

then show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

b) Evaluate by Green's theorem

$$\oint_C [(y - \sin x)dx + \cos x dy$$

where c is a triangle, where vertices are (0, 0), $\left(\frac{\pi}{2}, 0\right)$

and $\left(\frac{\pi}{2} + 1\right)$. 5

PART - II (20 Marks)

Answer ***any two*** questions.

6. a) Find the polar equation of the ellipse $\frac{x^2}{64}+\frac{y^2}{28}=1$; if the pole be at its right hand focus and positive direction of the x-axis be the positive direction of the polar axis. 5

b) PS P' is a focal chord of the conic. Prove that the angle between the tangents at P and P' is $\tan^{-1}\frac{2e\sin\alpha}{1-e^2}$, where '$\alpha$' is the angle between the chord and the major axis. 5

7. a) Reduce the following equation to its canonical form and determine the nature of the conic.

$$4x^2+4xy+y^2-12x-6y+5=0.$$ 5

b) Find the equation of the sphere which touches the sphere at the point (1, 1, –1) and passes through the origin. 5

8. a) Find the directional derivatives of

$$f(x, y, z)=x^2yz+4xz^2$$

at the point (1, 2, –1) in the direction of the vector

$$2\hat{i}-\hat{j}-2\hat{k}.$$ 5

but the repeated limits do not exist. 5

5. a) Prove that $\lim_{(x,y)\to(0,0)}\frac{x^4+y^4}{x^2+y^2}=0$

b) Determine the values of a, b, c so that

$$\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}\to 2 \text{ as } x\to 0$$ $2\frac{1}{2}+2\frac{1}{2}$