Ex/ED/Math/2.1/1/56/2019(OLD)

BACHELOR OF SCIENCE EXAMINATION, 2019(OLD)

(2nd Year, 2nd Semester)

MATHEMATICS

MATHEMATICS - I

ED - 2.1.1

Time : Two hours Full Marks : 50

Use a separate Answer-Script for each part

PART - I (15 Marks)

Symbols and notations have their usual meanings.

Answer ***any three*** questions.

1. Of A, B, C bu, any three subjects of a set, then prove that

   $A \times (B \cup C) = (A \times B) \bigcup (A \times C)$ 5

2. Define an equivalence relation on a set S.

   Suppose I bu, a defined by a & b if and only if (a-b) is divdsible bay 6. Then prove that () is an equuvalence relation.

   5

3. Define one-one and onto mapping with two example of each category.

   Let f : $R \rightarrow R$ and g:$R \rightarrow R$ bu two mapping defined by f(x)=x^2 and g(x)=x+3, for all x t R, Show that $fog \neq gof$. 5

4. Define group. Prove that z_4 the classes of residence of integers modulo 4, forms an () group with respect to ‘t’ addition (module 4) 5

5. Let () and equivalence relation on the set A, Then prove that fol all a, b, c A

cl(a)= cl(b) if and only if a b. 5

PART - III (15 Marks)

Answer ***any three*** questions.

6. Show that the function $f(x) = x\sin\frac{1}{x}, x \neq 0 = 0, x = 0$ in continous at x=0 5

7. A function f(x) is defined in [0,2] by

$f(x) = x^2 + x, 0 \leq x < 1$

$= 2, x = 1$

$= 2x^3 - x + 1, 1 < x \leq 2$

Examine differentiability of f(x) at x=1. 5

8. If $y = \sin\left(m\sin^{-1} n\right)$ show that

i) $(1-x^2)y_2 - xy_1 + m^2y = 0$

ii) $(1-x^2)y_{n+2} - (2n+1)\, xy_{n+1} - (n^2 - m^2)y_n = 0$ 2+3

9. State and prove Canchy’s Mean value thorem.

10. Expand sin x in a finite series in powers of x, with remainder in lagrange’ s form. 5

PART - II (20 Marks)

Answer ***any two*** questions.

6. a) Find the polar equation of the ellipse $\frac{x^2}{64}+\frac{y^2}{28}=1$; if the pole be at its right hand focus and positive direction of the x-axis be the positive direction of the polar axis. 5

b) PS P' is a focal chord of the conic. Prove that the angle between the tangents at P and P' is $\tan^{-1}\frac{2e\sin\alpha}{1-e^2}$, where '$\alpha$' is the angle between the chord and the major axis. 5

7. a) Reduce the following equation to its canonical form and determine the nature of the conic.

$$4x^2+4xy+y^2-12x-6y+5=0. \qquad 5$$

b) Find the equation of the sphere which touches the sphere at the point (1, 1, –1) and passes through the origin. 5

8. a) Find the directional derivatives of

$$f(x, y, z) = x^2yz+4xz^2$$

at the point (1, 2, –1) in the direction of the vector

$$2\hat{i}-\hat{j}-2\hat{k}. \qquad 5$$