7. (a) Using first MVT of integral calculus prove that

$$\frac{1}{4} \le \int_0^{1/4} \frac{dx}{\sqrt{1-x^2}} \le \frac{1}{\sqrt{15}}$$

(b) Let the function f be defined by

$$f(x) = \begin{cases} x^m \sin\frac{1}{x}, & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases}$$

then show that f is derivable at $x = 0$. Also determine m, when $f'(x)$ is continuous at $x = 0$. 5+5

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Ex/MATH/H/22/4.3/2019(OLD)

**BACHELOR OF SCIENCE EXAMINATION, 2019**

(2nd Year, 2nd Semester, Old Syllabus)

**MATHEMATICS (HONOURS)**

**Analysis - II**

**Paper : 4.3**

Time : Two hours Full Marks : 50

Symbols and Notations have their usual meanings.

Answer any ***five*** questions.

1. (a) Suppose f be derivable in [a,b] such that $f'(a) \neq f'(b)$ and r be any number between $f'(a)$ and $f'(b)$. Prove that, there exists $c \in [a,b]$ such that $f'(c) = r$.

(b) Evaluate :

(i) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

(ii) Find a and b such that

$$\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3} = 1$$ 5+5

2. (a) Let f : [a,b] → $\mathbb{R}$ be bounded function which has infinite number of points of discontinuities in [a,b] such that the set of all points of discontinuities of f in [a.b] has finite number of limit points. Show that f is Riemann integrable on [a,b].

(Turn Over)

(b) Suppose f be a function defined as

$$f(x)=\begin{cases}\dfrac{1}{2^n}, & when\ \dfrac{1}{2^{n+1}} < x \le \dfrac{1}{2^n},\\ 0, & when\ x=0 \quad n=0,1,2,3,....\end{cases}$$

Is f Riemann integrable on $[0,1]$? – Justify. 5+5

3. (a) Use Dirichlet's test to prove that

$$\int_0^\infty \frac{\sin x}{x}\,dx \text{ is convergent}$$

(b) Test the convergence of

(i) $\displaystyle\int_1^\infty \frac{x\tan^{-1}x}{\sqrt[3]{1+x^4}}\,dx$

(ii) $\displaystyle\int_1^\infty \left(\frac{\sin x}{x}\right)^2 dx$ 4+6

4. (a) Show that

$$\int_0^\infty \frac{\sin ax \sin bx}{x}\,dx = \frac{1}{2}\log\left(\frac{a+b}{a-b}\right), (a>b>0)$$



(b) Show that the integral $\displaystyle\int_0^1 x^{m-1}(1-x)^{n-1} \log x\,dx$ is convergent if $m>0$, $n<-1$. 4+6

5. (a) Prove that a function $f:[a,b]\to\mathbb{R}$ is of bounded variation on [a,b] if and only if f(x) can be expressed as the difference of two increasing functions. Is a function $f:[a,b]\to\mathbb{R}$ of bounded variation, R-integrable on [a,b]? Justify your answer.

(b) The function $f:[0,1]\to\mathbb{R}$ is defined by

$$f(x)=\begin{cases} x\sin\left(\pi/x\right) & if\ x\in[0,1]\\ 0, & x=0\end{cases}$$

Is f bounded variation on $[0,1]$? 6+4

6. (a) Let f(x+y) = f(x) . f(y), $\forall$ x, y $\in$ and f be derivable at $x=0$. If $f'(0)=l$ and $f(c)=m\neq 0$, then show that f is derivable at $c\in\mathbb{R}$ and $f'(c)=lm$.

(b) Let P and $P'$ be two partitions of [a,b] and $P'$ is a refinement of P. If $f:[a,b]\to\mathbb{R}$ is a function of bounded variation on [a,b], then prove that

$$V(P,f)\le V(P',f)$$ 5+5